Mathematics For Everyday Use

MATHEMATICS FOR EVERYDAY USE

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FOREWORD

WHETHER YOU ARE A SALESMAN, an office worker, or a mechanic, whether you are a typist, a private secretary, or a housewife, you cannot escape mathematics. We are literally immersed in it: income taxes, baseball averages, budgets, bills, social security, bank accounts, bargain sales, gasoline mileage, parcel post rates, bus fares. We need mathematics when repairing the house, buying a radio on time, operating the family car, or borrowing money; in deciding whether to bake a pie or buy one, whether to make Susan a dress or get her a ready-made frock, whether to paint the garage ourselves or hire a painter, whether to buy a new car or make the old one do another year, whether to buy more life insurance or another government bond. In short, the modern world is constantly counting, measuring and computing. The processes of mathematics are indispensable to the smooth working of the endless activities of civilized society.

Moderate skill in using elementary mathematics is not particularly difficult to achieve. It may even become an exciting experience, as we hope to be able to prove to you. This book is an informal, practical guide, intended to help you use your mathematics more often and more effectively. To this end, everyday experiences and problems of people everywhere have been drawn upon for illustrations; the necessary mathematics is carefully explained, and numerous typical problems are fully worked out to aid the reader.

Using mathematics can be a pleasure and a source of profit. Once you "feel at home" among the problems given here, you

will doubtless find many others of your own to which you can apply your skill. You will undoubtedly be surprised at the endless and unexpected applications that arise, and at the ease with which you can tackle them.

W. L. S.

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CHAPTER

PART I
Basic Elements of Arithmetic

ALTHOUGH MOST OF THE MATHEMATICS used in everyday life involves only *arithmetic,* both algebra and geometry are also used to some extent. In order to give the reader all the necessary groundwork, the first three chapters of this book (constituting Part I) afford a general review of the essential elements of arithmetic. In Chapters IV, V and VI (Part II), the essential elements of algebra and geometry are discussed. These first two parts together, therefore, provide all the skills needed to understand and apply mathematics to the problems occurring in everyday affairs, to which the rest of the book, Chapters VII–XXIV (Part III), is devoted.

If upon taking up this book you should feel that you are "a little weak" in the fundamental skills of arithmetic, quite apart from their application to actual problem situations, the material in Chapters I to III is intended to help you. If you are confident that you can perform ordinary computations with reasonable skill, then by all means pass on to Part II and begin at once with Chapter IV.

These first three chapters constitute a rapid review of the basic elements of arithmetic, i.e., computation, including the fundamental operations with whole numbers; the use of fractions, decimals, percentage, and ratio and proportion; and the basic principles of English and Metric measurements.

In addition to an explanation of the principles and methods involved, a number of practice problems have been included to help you master these basic principles should you need such practice. The answers to these problems are given at the end of the book.

CHAPTER I

MATHEMATICAL OPERATIONS WITH WHOLE NUMBERS

SOMEWHERE in the dim past, primitive man learned to count. At first by tallying, or matching physical objects in one group with the objects in a familiar group, he was enabled to count very crudely. Later, he developed number-names; then he used objects as counters. As time went on, he represented a *group* of objects by means of some other object or symbol. For example, a shepherd in counting his flock of thirty sheep might lay aside a pebble for each sheep as it passed by until he had "counted off" five; then he might accumulate another group of five, etc.; finally, he could count the number of groups of pebbles. Counting *groups* (instead of sheep or pebbles) is the basis of a number system. Our modern number system, of course, is based upon groups of ten instead of five, but the basic principle is no different. All the fundamental operations—addition, subtraction, multiplication and division—are based upon this same group concept.

In this chapter we will deal with the use of these four fundamental operations as applied to integers, or whole numbers. Although these basic operations were learned in early childhood, and may seem familiar, they have been discussed here again in the belief that such a review will be found helpful.

ADDITION

Addition Is Really Grouping

The operation of adding two numbers is essentially a process of grouping, or, more accurately, regrouping. When we add two numbers we do not increase anything; we *regroup* the numbers in accordance with the standard pattern or number system based on groups of ten. To illustrate: if we add 7 and 9 we "obtain" 16; but there has been no increase. We had the same number of objects before we "added" as afterward. What we've actually done is changed the arrangement of the two groups (7 and 9) into a new arrangement, the standard arrangement of 16 (i.e., 10 and 6). We *say* "sixteen," but we *think* "ten and six." And so it is for all combinations of numbers. This is the fundamental operation of addition—a process of very great convenience.

Basic Combinations in Addition

The business of adding numbers is enormously facilitated by knowing automatically certain basic combinations of the ten digits: 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. Taking them two at a time, and considering the arrangements obtained by reversing the pairs as separate combinations, we find that there are exactly 100 "addition combinations" possible. They have been set down here for reference.

1	2	3	4	5	6	7	8	9	0
0	0	0	0	0	0	0	0	0	0
—	—	—	—	—	—	—	—	—	—
1	2	3	4	5	6	7	8	9	0
1	1	1	1	1	1	1	1	1	1
—	—	—	—	—	—	—	—	—	—
1	2	3	4	5	6	7	8	9	0
2	2	2	2	2	2	2	2	2	2
—	—	—	—	—	—	—	—	—	—
1	2	3	4	5	6	7	8	9	0
3	3	3	3	3	3	3	3	3	3
—	—	—	—	—	—	—	—	—	—

1	2	3	4	5	6	7	8	9	0
4	4	4	4	4	4	4	4	4	4
1	2	3	4	5	6	7	8	9	0
5	5	5	5	5	5	5	5	5	5
1	2	3	4	5	6	7	8	9	0
6	6	6	6	6	6	6	6	6	6
1	2	3	4	5	6	7	8	9	0
7	7	7	7	7	7	7	7	7	7
1	2	3	4	5	6	7	8	9	0
8	8	8	8	8	8	8	8	8	8
1	2	3	4	5	6	7	8	9	0
9	9	9	9	9	9	9	9	9	9

These 100 basic number combinations must be thoroughly mastered before any progress in arithmetic can be made.

In order to develop further skill in addition, we must also learn to combine *two-figure* numbers with digits. Thus we must be able to combine 29 and 7 *directly* by *thinking* "29 and 7 give 36"; or "44 and 9 are 53"; etc. That is, in adding 58 and 6, we must not think "8 and 6 are 14, carry 1; 5 and 1 are 6; 64"; instead we must think "58 and 6 are 64" directly. When we can do this rapidly, we are ready to do more extensive addition.

Adding Columns

Ordinarily, when we have many numbers to add, they are usually written in vertical columns, one under another. We add the digits in each column separately, making use of what was mentioned in the preceding paragraph. The simplest and most direct way is to add the digits, *singly,* as we come to them. For example, in adding column (a), going from top to bottom, we would think: "5, 8, 10, 17, 18, 24, 28, 36, 39, 48, 52." Or, from bottom to top, column (b): "4, 13, 16, 24, 28, 34, 35, 42, 44, 47, 52."

(a)		(b)	
5		5	
3	8	3	10
2	10	2	
7	17	7	18
1	18	1	
6	24	6	28
4	28	4	
8	36	8	
3	39	3	48
9	48	9	
4		4	
52		52	

Some folks prefer to group the digits mentally into tens where possible: for example, in adding this column down, some might think: "10, 18, 28, 48, 52"; this, of course, is more rapid, but requires considerable skill and practice. For extra practice you might try these.

PROBLEMS FOR PRACTICE: NO. 1

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
8	3	9	3	6	4	9	2	5	7
5	6	4	2	2	3	2	5	6	3
1	3	5	8	8	4	3	8	4	8
3	8	6	7	1	2	8	0	2	6
8	5	2	8	5	6	9	8	9	6
6	9	1	6	2	3	6	7	4	1
2	1	9	7	9	8	4	6	4	3
5	7	4	8	7	1	9	4	6	0
4	2	5	7	8	3	7	2	1	8
8	8	8	8	2	9	4	3	8	6
3	5	4	1	5	2	0	4	7	3
6	4	2	3	2	5	6	2	1	8

Adding Several Columns

This simply requires a repetition of the above process, with the additional task of "carrying" a figure from one column to the next. The figure to be carried may be written as shown under (a) or (b), or, with practice, may be carried mentally as in (c).

(*a*)	(*b*)	(*c*)
449	449	449
232	232	232
871	871	871
356	356	356
384	384	384
653	653	653
25	$29^{3}4^{2}5$	2945
32		
26		
2945		

Checking Your Work

All additions should be checked. The simplest and most effective way to do this is to add the columns down from the top, until the sum has been found; then cover up the total and add the columns over again, this time from the bottom up, and compare the second sum with the first. If they agree, the work is almost certainly correct. (Some people add a column of figures two or three times and always get a different result; this is usually due to lack of practice and nothing else.) Naturally, when adding a column once *down* and again *up*, different combinations of figures will be selected; the chances of making counterbalancing errors are practically negligible. On the other hand, if a column is added in the same direction twice in rapid succession, a mistake occurring the first time may very pos-

Adding Long Columns

When it is necessary to add a rather long column of figures one or the other of two simple devices may be used, as shown in the following example:

(a)		(b)
3527		3527
8491		8491
4956		4956
5635		5635
4028		4028
6191		6191
3864	36,692	3864
8251		8251
4685		4685
2894		2894
4113		4113
5038		5038
6904	31,885	6904
	68,577	57
		62
		59
		62
		68,577

(a) Separate the column into two parts, add each part separately, and then combine the results; or
(b) Add each single column (units, tens, hundreds, etc.) separately, writing the sum below, and then add the partial sums together.

Of course, in actual business practice an adding machine is frequently used when long additions are required. For most people, the use of a machine is doubtless "easier"; on the other hand, there are certain disadvantages in using one. While the machine doesn't make "mistakes," the operator may punch the wrong key. Even if he doesn't, it generally takes longer by

machine than without it; experienced computers can add quite rapidly, and so can you with practice. Besides, an adding machine may not always be conveniently at hand just when you need it.

PROBLEMS FOR PRACTICE: NO. 4

(1)	(2)	(3)	(4)
$87.96	84,351	$ 864.83	$39,764.71
16.74	93,247	4118.40	76,420.28
17.89	62,158	3261.08	42,982.44
95.03	85,034	92.55	21,637.62
42.43	17,192	148.62	98,492.83
86.25	38,360	8913.48	74,094.04
97.85	14,384	672.94	36,341.26
85.05	26,125	2234.34	45,882.37
61.30	72,767	85.27	18,359.11
39.21	39,808	3416.09	54,628.98
83.47	81,464	2718.14	66,206.81
17.56	45,135	5336.76	17,134.23

Horizontal Addition

Cashiers, bookkeepers and accountants frequently find it very useful to add figures horizontally instead of vertically without first copying them over to rearrange them in columns instead of in rows. Proficiency in adding across rapidly and accurately should be cultivated as well as skill in vertical addition. It is of service in studying comparative records, statistical tabulations, annual statements, sales records, accounts, and so on.

PROBLEMS FOR PRACTICE: NO. 5

(1)	(2)	(3)
5+8+3+6=?	2+9+7+4=?	4+8+6+5+4=?
4+2+9+3=?	8+3+1+5=?	8+9+1+2+7=?
7+1+2+8=?	6+6+5+9=?	1+3+5+6+8=?
3+6+5+4=?	3+8+4+7=?	5+4+8+2+9=?
?+?+?+?=?	?+?+?+?=?	?+?+?+?+?=?

Easy Subtraction

Just in case you need a little practice in easy subtraction, the following exercises will furnish it. They do not involve zeros, nor do they require the operation of "borrowing."

PROBLEMS FOR PRACTICE: NO. 6

Subtract the following:

1.	56	83	92	74	68	99	85	78
	34	41	82	52	43	46	61	53

2.	697	835	792	654	885	798
	352	514	621	632	574	237
3.	778	694	895	946	855	793
	167	352	493	812	635	511

Subtraction Requiring Harder Combinations

We now come to subtraction involving *two-figure* combinations, like 7 from 16, 8 from 14, 5 from 13, etc.

PROBLEMS FOR PRACTICE: NO. 7

1.	154	136	147	189	165	138
	82	74	63	92	81	56

2.	1378	1836	1258	1597	1748
	856	904	953	706	918
3.	1569	1466	1687	1395	1498
	927	541	764	992	875

Various Methods of Subtracting

At least five different, well recognized methods may be used when subtracting. Not to confuse the reader, we shall explain only three of them—including the two most commonly used. In any case, continue to use the method of subtraction you are already accustomed to; it is generally inadvisable to change, unless you find another way easier.

Subtraction by the "Take-Away-Borrow" Method

This is the method probably used by three out of four people. It seems to be "natural" to subtract this way. At any rate, it is a very old method which was brought to Europe from India hundreds of years ago. When we subtract by "taking away and borrowing," we think:

75042 18578 56464	8 from 12=4 7 from 13=6 5 from 9=4 8 from 14=6 1 from 6=5

Subtraction by the "Take-Away-Carry" Method

This, too, is very old, and at one time, curiously enough, was practically the only method in use in this country. While it is theoretically a little better than the method described above, not many people appear to use it today. In using this method we think:

75042 18578 56464	8 from 12=4 8 from 14=6 6 from 10=4 9 from 15=6 2 from 7=5

Subtraction by the "Addition-Carry" Method

This method differs from both of the above in that we do not "take-away," but *think* as though we were adding, thus:

75042 18578 56464	8 and what make 12? (4) 8 and what make 14? (6) 6 and what make 10? (4) 9 and what make 15? (6) 2 and what make 7? (5)

This is also known as the "Austrian method" of subtraction, and has been widely taught in American schools in recent years.

Its advocates claim several advantages over the other two methods, but its superiority is somewhat a matter of opinion.

Checking Subtraction

The simplest method of checking subtraction, of course, is to add the remainder to the subtrahend; if this sum equals the minuend, the work is correct. The beginner may wish at first to *write out* the sum so obtained, placing it beneath the remainder as shown; this ought not to become a habit, however; with practice the addition is performed mentally and "checked" against the original minuend. Checking all subtractions conscientiously should become an automatic habit. *Never omit the check!*

$$\boxed{\begin{array}{r} 935{,}806 \\ 582{,}689 \\ \hline 353{,}117 \\ \hline 935{,}806 \end{array}}$$

PROBLEMS FOR PRACTICE: NO. 8

Subtract by whichever method you prefer or are used to, and check your result mentally:

(1)	(2)	(3)	(4)
483,694	736,805	954,832	698,321
358,725	584,962	790,536	625,747

(5)	(6)	(7)	(8)
$2946.80	$8106.59	$872,659.18	$682,418.25
786.52	352.75	493,122.63	75,603.44

(9) A total of 1,214,958 passengers were carried by all air lines in the U. S. in a recent year as compared with 1,063,480 the year before. What was the increase?

(10) In a recent year the total Federal Income Tax collected amounted to $2,586,243,954; the preceding year the amount was $2,148,663,876. By how much did it increase?

Horizontal Subtraction

In working with business records it is occasionally convenient to subtract *horizontally,* i.e., to find the difference between two numbers that are written next to each other on the same horizontal line. Sometimes the smaller of the two numbers may be placed to the left, sometimes to the right of the larger. Skill in subtracting such numbers mentally without first *copying* them over and writing them one under the other is quite useful; it saves considerable time and minimizes errors due to copying. Such subtraction often arises in connection with bank statements, checkbook stubs, sales records, payrolls, balance sheets, profit and loss statements, and various statistical tabulations. Horizontal subtraction can sometimes be conveniently checked by column addition, as for example:

RECORD OF DAILY SALES

Clerk	*Gross Receipts*	*Returns and Allowances*	*Net Sales*
Brown	\$142.15	\$22.30	\$119.85
Purdy	210.74	19.86	190.88
Smith	189.16	5.72	183.44
Clayton	253.99	34.11	219.88
Harris	204.43	27.64	176.79
TOTALS:	\$1000.47	\$109.63	\$890.84

PROBLEMS FOR PRACTICE: NO. 9

(1)	(2)	(3)
532—176=?	864—356=?	911—374=?
328—214=?	672—433=?	500—238=?
709—422=?	920—657=?	798—560=?
? — ? = ?	? — ? = ?	? — ? = ?

(4)

Receipts	*Disbursements*	*Balance*
$362.14	$ 75.00	?
584.77	122.35	?
627.50	96.87	?
483.22	155.10	?
504.18	108.92	?
?	?	?

(5)

Deposits	*Withdrawals*	*Balance*
$114.16	$ 33.50	?
95.28	18.75	?
141.70	56.19	?
200.50	40.75	?
175.25	128.10	?
130.44	25.95	?
?	?	?

MULTIPLICATION

Abbreviated Addition

Consider the problem of adding 36 seven times. This can be shortened by multiplying as follows:

$$\begin{array}{r} 36 \\ \times 7 \\ \hline 42 \\ 21 \\ \hline 252 \end{array} \qquad\qquad \begin{array}{r} 36 \\ 36 \\ 36 \\ 36 \\ 36 \\ 36 \\ 36 \\ \hline 252 \end{array}$$

The abbreviation would have been more noticeable had we added 36 sevens, or *taken* 7 thirty-six times, thus: 7+7+7+7+ .... to 36 terms. The real convenience in abbreviated addition, or multiplication, becomes apparent when dealing with larger numbers, e.g., 197×213; by actual addition the procedure would be so impractical as to be prohibitive.

The result of the multiplication of two or more numbers is called their *product;* the numbers multiplied together are known as the *factors*. The order in which they are multiplied is immaterial, i.e., 57×82 gives the same product as 82×57.

Including the zeros, there are 90 basic "multiplication facts," or arrangements of digits taken two at a time, counting reversals as separate "facts." These must be committed to memory until they are automatic if multiplications are to be carried out with facility. Actually, it is also desirable to know the multiplication table beyond "9×9," at least up to "12×12." Speed and accuracy in general multiplication with larger numbers depends upon skill in using the multiplication facts. Ordinary, simple multiplication is carried out as shown in (a), although actually the work is shortened by doing the "carrying" mentally as shown in (b):

(*a*)

```
 7528
    8
-----
   64
  16
 40
56
-----
60224
```

(*b*)

```
  7528
     8
------
60,224
```

More Extensive Multiplication

By extending the above procedure, the multiplication of larger numbers is readily carried out; the work is checked by reversing the order of multiplication.

EXAMPLE

Multiply:

```
     2485
      764
---------
     9940
   14910
  17395
---------
1,898,540
```

Check:

```
      764
     2485
---------
     3820
    6112
   3056
  1528
---------
1,898,540
```

PROBLEMS FOR PRACTICE: NO. 10

Multiply each of the following, and check by reversing the order of multiplication:

1. 354 × 67	*2.* 893 × 145	*3.* 628 × 236	*4.* 532 × 728
5. 2135 × 264	*6.* 4908 × 329	*7.* 56,394 × 144	
8. $396.50 × 29	*9.* $156.75 × 132	*10.* $27.14 × 365	

Accuracy and Estimating the Answer

When multiplying decimals, the process is precisely the same, care being taken to have the decimal point in the right place in the product. *The number of decimal places in the product must be the sum of the number of decimal places in the two factors multiplied;* e.g.:

EXAMPLE

```
   48.71   (2 decimal places)
    36.4   (1 decimal place)
  ------
   19484
  29226
 14613
 -------
1773.044   (3 decimal places)
```

A good habit to get into is that of estimating the product before actually multiplying. In this way careless mistakes will be avoided, especially where decimals are involved. For example, the cost of 24 gallons of gasoline at 16.4¢ a gallon would be estimated at somewhere around $4; by actual multiplication 24×$.164=$3.936, or $3.94. By estimating the answer before actually multiplying, foolish mistakes such as saying $39.36, or $.39, would be avoided.

Short Cuts in Multiplication

There are a few convenient short methods in multiplication which everyone ought to know, as they make it possible to multiply easily and quickly without using pencil and paper.

EXAMPLES

1. To multiply a number by 10,000, 1000, etc., move the decimal point to the *right* as many places as there are terminal zeros in the multiplier.

$$9.82 \times 1000 = 9820$$
$$45.863 \times 100 = 4586.3$$

2. To multiply a number by 0.1, 0.01, 0.001, etc., move the decimal point to the *left* as many places as there are terminal zeros in the multiplier.

$$47.3 \times 0.001 = 0.0473$$
$$2915 \times 0.01 = 29.15$$

3. To multiply a number by 50, move the decimal point two places to the right and divide by 2.

$$483 \times 50 = 48300 \div 2 = 24150$$
$$67.9 \times 50 = 6790 \div 2 = 3395$$

4. To multiply a number by 25, move the decimal point two places to the right and divide by 4.

$$364 \times 25 = 36400 \div 4 = 9100$$
$$56.28 \times 25 = 5628 \div 4 = 1407$$

Aliquot Parts

If a number is contained in another number a whole number of times, the first number is said to be an *aliquot part* of the second number. Thus, 10¢, 25¢, and 33⅓¢ are all aliquot parts of \$1, since they are contained 10, 4, and 3 times, respectively, in \$1. Some numbers, while not aliquot parts themselves, are nevertheless convenient multiples of aliquot parts; e.g., 37½¢, 66⅔¢, and 75¢ are multiples of ⅛, ⅓, and ¼ of \$1, respec-

tively. The aliquot parts which follow are particularly helpful and should by all means be memorized for convenience. When thoroughly mastered so that they are recognized immediately, they can be used to advantage in multiplication examples where one of the factors is an aliquot part of a number, thus saving time and diminishing the probability of error.

COMMON ALIQUOT PARTS OF $1,

Together with Their Multiples

50¢=$½	12½¢=$⅛	33⅓¢=$⅓	8⅓¢=$1/12
25¢=$¼	37½¢=$⅜	66⅔¢=$⅔	6¼¢=$1/16
75¢=$¾	62½¢=$⅝	16⅔¢=$⅙	
20¢=$⅕	87½¢=$⅞	83⅓¢=$⅚	

Extensions on an Invoice

One of the many practical uses of multiplication is in finding the total costs for the various items on an invoice or a bill of goods. This is called "making the extensions."

EXAMPLE

MONARCH DEPARTMENT STORE
55 Main St., New Rochelle, N. Y.

Aug. 28, 194–

Sold To:

Mrs. Harry Nichols,
209 Clinton St.

Aug. 4	8 shade rollers	@ 79¢	6	32
10	4 pr. curtains	@ 1.98	7	92
12	1½ doz. glasses	@ 1.84	2	76
19	2 cans floor wax	@ 1.19	2	38
			19	38

Aliquot parts are often convenient in making extensions, saving time and effort.

EXAMPLES

Find the cost of each of the following:

$$\begin{aligned}
24 \text{ lb. @ } 33\tfrac{1}{3}¢ &= 24\times\$\tfrac{1}{3}=\$\ 8.00\\
48 \text{ lb. @ } 25¢ &= 48\times\$\tfrac{1}{4}=\$12.00\\
64 \text{ lb. @ } 37\tfrac{1}{2}¢ &= 64\times\$\tfrac{3}{8}=\$24.00\\
36 \text{ lb. @ } 75¢ &= 36\times\$\tfrac{3}{4}=\$27.00
\end{aligned}$$

Furthermore, when using aliquot parts, it is sometimes helpful to *interchange* the quantity and the unit price:

$$\begin{aligned}
12\tfrac{1}{2} \text{ lb. @ } 72¢ &= \$\ \tfrac{1}{8}\times 72=\$\ 9\\
75 \text{ yd. @ } 36¢ &= \$\ \tfrac{3}{4}\times 36=\$27\\
6\tfrac{1}{4} \text{ lb. @ } 32¢ &= \$\tfrac{1}{16}\times 32=\$\ 2
\end{aligned}$$

Another variation along these lines is to use aliquot parts of \$10, \$100, or \$1000 instead of aliquot parts of \$1.

1. Find the cost of 32 vacuum cleaners at \$62.50 each.

$$\$62.50 \text{ is } \tfrac{5}{8} \text{ of } \$100, \text{ or } \frac{\$500}{8}$$

$$32\times\frac{\$500}{8}=\$2000, \textit{ Ans.}$$

2. What is the value of 56 sewing machines at \$125 each?

$$\$125 \text{ is } \tfrac{1}{8} \text{ of } \$1000, \text{ or } \frac{\$1000}{8}$$

$$56\times\frac{\$1000}{8}=\$7000, \textit{ Ans.}$$

PROBLEMS FOR PRACTICE: NO. 11

Find the following products, using aliquot parts where convenient; estimate your answer beforehand:

1. 64 lb. @ $37\frac{1}{2}$¢	4. 144 @ $62\frac{1}{2}$¢	7. 360 @ \$7.50
2. $62\frac{1}{2}$ ft. @ 48¢	5. 2400 @ $8\frac{1}{3}$¢	8. 960 @ \$12.50
3. $66\frac{2}{3}$ yd. @ \$1.29	6. 96 @ \$37.50	9. 1440 @ \$1.25

10. What is the cost of painting 4280 sq. ft. at 3½¢ a square foot?
11. What is the cost of 960 cu. ft. of gas at $1.12½ per 1000 cu. ft.?
12. Find the amount of a disability claim of $22.75 per week for 38 weeks.

DIVISION

The Inverse of Multiplication

Two factors when multiplied together give a certain product; if that product is divided by either factor, the other factor is called the *quotient*. The product is referred to in this instance as the *dividend*, and the factor by which we divide is called the *divisor*. Division is thus the reverse of multiplication. Any product is divisible "evenly" by any of its factors. Or, putting it another way, division is the process of finding out how many times one number "is contained" in another.

If a divisor is not contained an exact (integral) number of times in the dividend, the amount "left over" is called the remainder, and may be expressed either as a common fraction or as a decimal.

Simple Short Division

This is the procedure used when dividing by numbers small enough to permit doing the division mentally, as, for example, when dividing by a digit, by 10, by 11, or even 12.

EXAMPLES

1. Divide 26,349 by 8.

$$8\,\overline{)26{,}349}$$
$$3{,}293;\ \textit{remainder}\ 5.$$

or $3293\frac{5}{8}$, *Ans.*

2. Divide 4638 by 12:

$$12\,\overline{)4638}$$
$$386;\ \textit{remainder}\ 6.$$

or $386\frac{6}{12}=386\frac{1}{2}$, *Ans.*

Long Division

If the divisor is greater than 12, the method of long division is used. Not only is the arrangement slightly different in this case, but also "partial quotients" must be estimated mentally by trial.

EXAMPLES

1. Divide 28,463 by 17.

```
        1674
    --------
17  )28463
     17
     ---
     114
     102
     ----
      126
      119
      ----
        73
        68
        --
         5 (Remainder)
```

Ans.: $1674\,{}^{5}/_{17}$

2. Divide 968,395 by 294.

```
          3293
      --------
294  )968395
      882
      ---
       863
       588
       ----
       2759
       2646
       -----
        1135
         882
       -----
         253 (Remainder)
```

Ans.: $3293\,{}^{253}/_{294}$

PROBLEMS FOR PRACTICE: NO. 12

Divide the following, using short division:

1. $9314 \div 7$	3. $45{,}932 \div 8$	5. $374{,}007 \div 6$
2. $8632 \div 9$	4. $66{,}304 \div 11$	6. $853{,}291 \div 12$

Divide the following, using long division:

7. $4685 \div 43$	9. $76{,}483 \div 328$
8. $31{,}682 \div 19$	10. $498{,}642 \div 144$

Checking Division

The simplest and best way of checking the accuracy of division is to multiply the quotient obtained by the divisor used, and then add the remainder, if any; if the result so obtained equals the original dividend, the division was correctly performed.

EXAMPLE

Divide, and check: 43,192÷23.

$$\begin{array}{rl} & 1877 \\ 23 & \overline{)43192} \\ & 23 \\ \hline & 201 \\ & 184 \\ \hline & 179 \\ & 161 \\ \hline & 182 \\ & 161 \\ \hline & 21 \ (\textit{Remainder}) \end{array} \qquad\qquad \begin{array}{r} \underline{\text{Check:}} \\ 1877 \\ 23 \\ \hline 5631 \\ 3754 \\ \hline 43171 \\ 21 \\ \hline 43192 \end{array}$$

The Nearest Tenth or Hundredth

For most practical purposes, when a division does not come out exact (which is more often than not), the remainder is expressed as a decimal rather than as a common fraction. The division may then be carried to one, two, or more decimal places, i. e., to any degree of accuracy desired. It is then said to be "correct to the nearest tenth," or the nearest hundredth, etc.

EXAMPLES

1. Find, correct to the nearest hundredth, 82,597÷12.

$$\begin{array}{rl} 12 & \underline{)82597.00} \\ & 6883.08, \ \textit{Ans.} \end{array}$$

2. Find, correct to the nearest thousandth, $762 \div 39$.

$$
\begin{array}{l}
\;19.538, \textit{ Ans.} \\
39\,\overline{)762.000} \\
\;\underline{39} \\
\;372 \\
\;\underline{351} \\
\;210 \\
\;\underline{195} \\
\;150 \\
\;\underline{117} \\
\;330 \\
\;\underline{312}
\end{array}
$$

Tests of Divisibility

It is sometimes desirable to tell by inspection whether a number is exactly divisible by some other number. For this purpose certain tests of divisibility are helpful; they are listed here for reference.

1. A number is exactly divisible by 2:—only if its extreme right-hand digit is even.
2. A number is exactly divisible by 3:—only if the sum of its digits is divisible by 3.
3. A number is exactly divisible by 4:—only if the number expressed by the two right-hand digits is itself divisible by 4.
4. A number is exactly divisible by 5:—only if it ends in 0 or 5.
5. A number is exactly divisible by 6:—only if it is even and the sum of its digits is divisible by 3.
6. A number is exactly divisible by 8:—only if the number expressed by the three right-hand digits is itself divisible by 8.
7. A number is exactly divisible by 9:—only if the sum of its digits is divisible by 9.

Estimating a Quotient

Just as in multiplication it was found useful to estimate the product before multiplying as an added caution against certain types of errors, notably misplacing the decimal point,—so in division, by a similar procedure, it is always wise to estimate the quotient beforehand in round numbers.

EXAMPLES

Estimate the quotient for each of the following:

(a) $714 \div 36$ (b) $4188 \div 58$

(a) $714 \div 36$ is approximately $700 \div 35$, or about 20. (By actual division the result is 19.8)

(b) $4188 \div 58$ is approximately $4200 \div 60$, or about 70. (By actual division the result is 72.2)

Short Cuts in Division

Only two or three simple convenient short cuts are available in division:

(I) To divide by 10, 100, 1000, etc.: move the decimal point one, two, three, etc., places to the left; or, move the decimal as many places to the left in the dividend as there are ciphers in the divisor.

Example: $69{,}348 \div 100 = 693.48$

(II) To divide by 25: move the decimal point two places to the left in the dividend and multiply the result by 4.

Example: $841 \div 25 = 8.41 \times 4 = 33.64$

(III) To divide by 50: move the decimal point two places to the left in the dividend and multiply the result by 2.

Example: $6574 \div 50 = 65.74 \times 2 = 131.48$

Averages

Among the many uses of division is included the finding of an average, or more technically, the arithmetic mean, which

is defined as the arithmetic sum of a number of values divided by the total number of values.

EXAMPLES

1. What is the average score on an eligibility examination of the six top ranking candidates whose scores were 88.2, 85.8, 82.1, 80.4, 79.1, respectively?

```
     88.2
     85.8
     82.1
     80.4
     79.1
   ------
 5 )415.6
   ------
     83.12, Ans.
```

2. A streamlined Diesel train covered a run of 656 miles in 6 hr. 24 min. What was its average speed per hour?

6 hr. 24 min.=6.4 hr.

```
      102.5
    -------
 64 )6560.0
     64
     --
      160
      128
      ---
       320
       320
       ---
```

Ans.: 102.5 mi. per hr.

PROBLEMS FOR PRACTICE: NO. 13

Find the result correct to the nearest tenth:

1. 954÷18   2. 698÷36   3. 4312÷95

Find the result correct to the nearest hundredth:

4. 734÷12   5. 3819÷52   6. 60,491÷125

7. A man drove his car a distance of 278 miles; if it took him 7¾ hr., what was his average speed per hour?

8. If there are 144 square inches in a square foot, how many square feet are there in a flat surface of 2800 square inches?
9. If the total rainfall in a certain city during each of six successive months was 2.2, 1.7, 3.4, 2.8, 1.9, and 2.5 inches, what was the average monthly rainfall during this six-month period?
10. If over a long period of years an insurance company paid $48,879,400 for 356,600 accident claims, what was the average amount of each claim?

CHAPTER II

MATHEMATICAL OPERATIONS WITH PARTS OF NUMBERS

THE GREEKS used a system by which whole numbers represented fractions, much as we designate time; e.g., 2 hours and 20 minutes (2:20) instead of $2\frac{1}{3}$ hours. This scheme (based on $\frac{1}{60}$) is known as the *sexagesimal* system; it was closely related to astronomy, and was used in measuring angles, even as we still do today. We say, for example, an angle of 22° 30′ 30″, rather than $22+\frac{1}{2}+\frac{1}{120}$ degrees.

Similarly, the Romans also avoided fractions as much as possible, preferring instead to use subdivisions of units (smaller units), which allowed them still to work with whole numbers. Thus they found it much easier to think of 4 *uncias* (one *uncia* $=\frac{1}{12}$ of a Roman pound) than to think of $\frac{1}{3}$ of a pound. For that matter, even today we usually say that a parcel to be mailed weighs 2 lb. 3 oz. rather than $2\frac{3}{16}$ lb., just as it is less awkward to say that a man is 5′ 10″ tall instead of $5\frac{5}{6}$ feet in height.

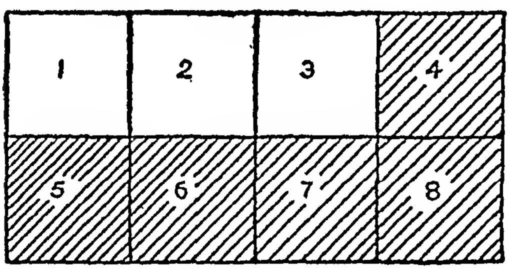
Much of the world's work involves decimals and fractions—the sciences, the practical arts, technology. Practically all business problems involve per cents—one of the most useful and indispensable devices of arithmetic. The use of decimals and

per cents is far more practical and convenient, in many cases, than common fractions.

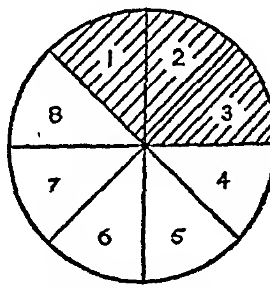
FRACTIONS

Common Fractions

If a number or a quantity is thought of as being broken up, or subdivided into any number of *equal* parts, and then a certain number of those parts is considered separately in relation to the total number of such parts, we arrive at the idea of a fraction. For example, consider the fraction $3/8$; it suggests that something (a dollar, a yard, a pound, a layer cake) has been divided, in our thinking or actually, into *eight equal parts,* and then that *three of those parts* are being considered separately.



$3/8$ AND $5/8$



$3/8$ AND $5/8$

Writing Fractions

Common fractions are written with a horizontal or a slanting line between the two numbers, as $\frac{3}{4}$ or $5/8$. The two numbers are called the *terms* of the fraction. The lower number is the *denominator,* and indicates the complete number of equal parts into which the whole quantity has been subdivided. The upper number is the *numerator,* and indicates how many of those equal parts are being considered.

Ordinarily the numerator of a fraction is smaller than its denominator. Such a fraction is called a *proper* fraction, and it denotes or represents *part of a whole* thing. Evidently, if we added the numerators of the two fractional parts of a quantity, like $\frac{1}{4}$ and $\frac{3}{4}=\frac{4}{4}$, or $\frac{3}{8}$ and $\frac{5}{8}=\frac{8}{8}$, we should have the whole quantity. In other words, a fraction whose numerator and denominator are equal, like $\frac{8}{8}$ or $\frac{5}{5}$, must be equal to 1, or an entire unit; although the whole quantity has been subdivided into a certain number of equal parts, we are nevertheless considering *all* of those parts, and therefore dealing with the *whole* quantity. Such a fraction, whose numerator equals its denominator, is called an *improper* fraction, suggesting that in a certain sense (not representing a *part* of something) it is not really a fraction at all, but merely has the *form* of a fraction.

Another kind of improper fraction is one whose numerator is *greater* than its denominator, as for example, $\frac{3}{2}$ or $\frac{7}{4}$. This kind of fraction results from the addition of a proper fraction to a whole number. Thus, a whole pie plus half a pie makes a pie and a half, or $1+\frac{1}{2}=\frac{2}{2}+\frac{1}{2}=\frac{3}{2}$. Or again, $1\frac{3}{4}$ lb. is $\frac{7}{4}$ lb.; i.e., $1+\frac{3}{4}=\frac{4}{4}+\frac{3}{4}=\frac{7}{4}$. When written as $1\frac{1}{2}$ or $1\frac{3}{4}$ we speak of "mixed numbers," which simply means the indicated addition of a whole number (integer) and a common fraction. Hence every improper fraction (except one like $\frac{3}{3}$ or $\frac{5}{5}$) can be expressed as a mixed number; likewise, every mixed number can also be written as an improper fraction. Thus:

$$\frac{6}{5}=\frac{5}{5}+\frac{1}{5}=1+\frac{1}{5}=1\frac{1}{5}$$

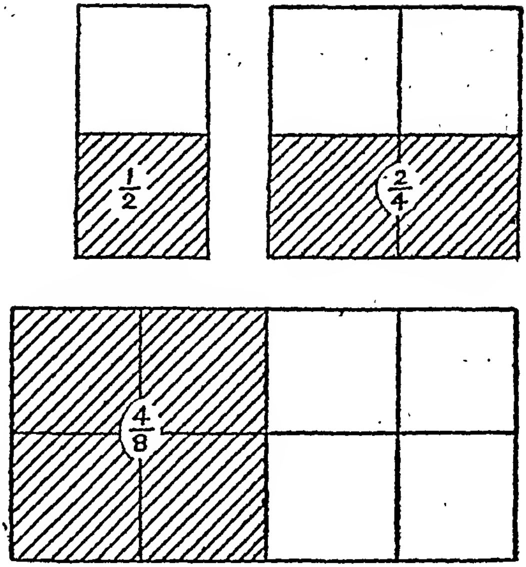
$$1\frac{2}{3}=1+\frac{2}{3}=\frac{3}{3}+\frac{2}{3}=\frac{5}{3}$$

Reducing Fractions

Experience suggests that taking 2 parts out of 4 equal parts is equivalent to taking 1 part out of 2 equal parts; or that taking 4 parts out of 8 parts is equivalent to taking 2 out of 4, etc., as indicated by the accompanying diagrams. This idea can be extended still further, as illustrated by the following:

$\frac{3}{4}=\frac{6}{8}=\frac{9}{12}$

$\frac{8}{12}=\frac{6}{9}=\frac{4}{6}=\frac{2}{3}$



If the numerator *and* the denominator of any given fraction are *both* multiplied, or divided, by the same number, the *value* of the fraction remains unchanged; i.e., the new fraction so obtained represents the same part of the entire quantity that the original fraction represented. When both terms of a fraction are divided by the same number until they cannot be further divided, the fraction is said to have been "reduced to lowest terms."

EXAMPLES

1. Reduce $\frac{15}{60}$ to lowest terms.

$$\frac{15}{60}=\frac{5}{20}=\frac{1}{4}, \textit{ Ans.}$$

2. Reduce $\frac{24}{72}$ to lowest terms.

$$\frac{24}{72}=\frac{12}{36}=\frac{6}{18}=\frac{3}{9}=\frac{1}{3}, \textit{ Ans.}$$

Similarly, as already mentioned, mixed numbers can be changed to improper fractions, and improper fractions can be reduced to mixed numbers; their values, however, are not changed in the process.

EXAMPLES

1. Reduce $^{27}/_4$ to a mixed number.

$$^{27}/_4={}^{24}/_4+{}^3/_4=6+{}^3/_4=6^3/_4,\ \textit{Ans.}$$

2. Change $8^2/_3$ to an improper fraction.

$$8^2/_3=8+{}^2/_3={}^{24}/_3+{}^2/_3={}^{26}/_3,\ \textit{Ans.}$$

PROBLEMS FOR PRACTICE: No. 14

Change each of the following to a mixed number:

1. $^{11}/_3$	2. $^{27}/_{14}$	3. $^{24}/_{20}$	4. $^{42}/_{18}$
5. $^{22}/_7$	6. $^{38}/_{15}$	7. $^{38}/_{12}$	8. $^{28}/_{16}$

Express each of the following as an improper fraction:

9. $10^3/_4$	10. $16^1/_4$	11. $22^1/_2$	12. $11^3/_5$
13. $12^2/_5$	14. $5^7/_8$	15. $3^3/_{16}$	16. $25^5/_6$

Addition of Fractions with a Common Denominator

Fractions having the same denominator, like $^1/_4$ and $^3/_4$, or $^3/_{16}$, $^5/_{16}$, and $^7/_{16}$, are called like fractions, or similar fractions; the denominator of each is called the common denominator, since it is the same for all the fractions. Such similar fractions, having a common denominator, are easily added; their numerators are merely added together, and the sum is divided by the common denominator, which gives the desired result. The result is usually reduced to lowest terms if possible.

That this procedure is reasonable may be seen from the following analogy:

$$^1/_{12}+{}^3/_{12}+{}^5/_{12}=\frac{1+3+5}{12}={}^9/_{12}$$

1 "twelfth"	1 "apple"
3 "twelfths"	3 "apples"
5 "twelfths"	5 "apples"
9 "twelfths"	9 "apples"

EXAMPLES

1. Add: $^3/_{16}+{}^5/_{16}+{}^9/_{16}+{}^7/_{16}$

$$^3/_{16}+{}^5/_{16}+{}^9/_{16}+{}^7/_{16}=\frac{3+5+9+7}{16}={}^{24}/_{16}={}^3/_2=1^1/_2,\ \textit{Ans.}$$

2. Add: $5\frac{1}{4}$
$7\frac{3}{4}$
$8\frac{1}{4}$
$\frac{1}{4}+\frac{3}{4}+\frac{1}{4}=\frac{5}{4}=1\frac{1}{4}$
$5+7+8=20$
$1\frac{1}{4}+20=21\frac{1}{4}$, *Ans.*

Addition of Unlike Fractions

When fractions have *different* denominators they are said to be unlike fractions, such as $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{3}{5}$. Before such fractions can be added, the terms must be changed so that they become like fractions; then they can be added as previously explained. The new denominator, to which all the fractions are reduced, is called the *lowest common denominator* (L.C.D.), and can usually be found by inspection or by trial.

EXAMPLES

1. Add: $\frac{5}{12}$, $\frac{2}{3}$, and $\frac{3}{4}$.
The L.C.D. of 12, 3, and 4 is seen to be 12.
$\frac{5}{12}=\frac{5}{12}$
$\frac{2}{3}=\frac{8}{12}$
$\frac{3}{4}=\frac{9}{12}$
$\frac{22}{12}=1\frac{10}{12}=1\frac{5}{6}$, *Ans.*

2. Add: $6\frac{2}{3}$, $9\frac{1}{2}$, and $10\frac{3}{5}$.
Here the L.C.D. is 30.
$6\frac{2}{3}=6\frac{20}{30}$
$9\frac{1}{2}=9\frac{15}{30}$
$10\frac{3}{5}=10\frac{18}{30}$
$25\frac{53}{30}=26\frac{23}{30}$, *Ans.*

Subtraction of Fractions and Mixed Numbers

The subtraction of one fraction from another, or one mixed number from another, is carried out in the same way as addition. If the fractions have a common denominator, subtract their numerators and divide the result by their common denom-

inator. If they are unlike fractions, change them so that they have a common denominator and proceed as before. When subtracting a mixed number from another, subtract the fractional parts first, as already explained; then continue as usual.

EXAMPLES

1. Subtract $\frac{3}{8}$ from $\frac{5}{12}$.

$$\begin{aligned} &\text{L.C.D.}=24 \\ &\tfrac{5}{12}=\tfrac{10}{24} \\ &\tfrac{3}{8}=\underline{\tfrac{9}{24}} \\ &\qquad \tfrac{1}{24}, \textit{ Ans.} \end{aligned}$$

2. From $14\frac{1}{6}$ subtract $8\frac{4}{5}$.

$$\begin{aligned} &\text{L.C.D.}=30 \\ &14\tfrac{1}{6}=14\tfrac{5}{30}=13\tfrac{35}{30} \\ &8\tfrac{4}{5}\qquad\quad\;\; =\underline{8\tfrac{24}{30}} \\ &\qquad\qquad\quad\;\; 5\tfrac{11}{30}, \textit{ Ans.} \end{aligned}$$

PROBLEMS FOR PRACTICE: NO. 15

Add the following:

1. $\frac{3}{8}+\frac{2}{3}$
2. $\frac{2}{5}+\frac{3}{4}$
3. $\frac{5}{12}+\frac{5}{8}$
4. $\frac{2}{3}+\frac{1}{2}+\frac{5}{6}$
5. $\frac{1}{6}+\frac{5}{12}+\frac{2}{3}$
6. $\frac{3}{8}+\frac{1}{2}+\frac{3}{4}$
7. $\frac{3}{8}+\frac{3}{2}+\frac{7}{6}$
8. $\frac{3}{2}+\frac{2}{5}+\frac{9}{10}$
9. $\frac{7}{8}+\frac{9}{16}+\frac{3}{4}+\frac{1}{2}$
10. $12\frac{3}{8}+16\frac{1}{4}$
11. $\frac{5}{12}+2\frac{1}{2}+8\frac{1}{4}$

Subtract the following:

12. $\frac{7}{8}-\frac{3}{4}$
13. $\frac{3}{4}-\frac{3}{8}$
14. $\frac{7}{12}-\frac{3}{8}$
15. $\frac{21}{25}-\frac{3}{5}$
16. $\frac{33}{40}-\frac{15}{20}$
17. $\frac{20}{3}-\frac{5}{6}$
18. $25\frac{7}{8}-18\frac{3}{4}$
19. $22\frac{1}{4}-5\frac{3}{8}$
20. $16\frac{1}{3}-8\frac{1}{2}$

Multiplication with Fractions

In order to multiply two or more fractions together, it is not necessary to reduce them to a common denominator first; instead, the process of cancellation is used, and the resulting numerators and denominators are respectively multiplied together.

EXAMPLES

1. Multiply: $\frac{3}{8}\times\frac{4}{9}\times\frac{5}{2}$.

$$\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{8}}}\times\frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{9}}}\times\frac{5}{2}=\frac{1\times1\times5}{2\times3\times2}=\frac{5}{12},\ \textit{Ans.}$$

2. Multiply: $\frac{5}{8}\times14\times3\frac{1}{7}$.

$$\frac{5}{\underset{\underset{2}{\cancel{4}}}{\cancel{8}}}\times\frac{\overset{\overset{1}{\cancel{2}}}{\cancel{14}}}{1}\times\frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}}=\frac{5\times1\times11}{2\times1\times1}=\frac{55}{2}=27\frac{1}{2},\ \textit{Ans.}$$

Division with Fractions

Since division is the opposite, or inverse of multiplication, a little reflection will show that dividing by 2 is equivalent to multiplying by $\frac{1}{2}$; or dividing by 8 is equivalent to multiplying by $\frac{1}{8}$. Now since 2 is the same as $\frac{2}{1}$, then $\frac{1}{2}$ is the fraction obtained by *inverting* $\frac{2}{1}$; each is called the *reciprocal* of the other. Thus the reciprocal of $\frac{1}{4}$ is 4; of 8, $\frac{1}{8}$; of $\frac{2}{3}$, $\frac{3}{2}$; of $\frac{3}{2}$, $\frac{2}{3}$; etc. Therefore, to divide by a fraction, we simply invert the fraction and multiply by it instead.

EXAMPLES

1. Divide 27 by $\frac{3}{4}$.

$$27\div\frac{3}{4}=27\times\frac{4}{3}=36,\ \textit{Ans.}$$

2. Divide $3\frac{3}{4}$ by $6\frac{1}{2}$.

$$3\frac{3}{4}\div6\frac{1}{2}=\frac{15}{4}\div\frac{13}{2}=\frac{15}{\underset{2}{\cancel{4}}}\times\frac{\cancel{2}}{13}=\frac{15}{26},\ \textit{Ans.}$$

PROBLEMS FOR PRACTICE: NO. 16

Multiply:

1. $\frac{5}{6}\times\frac{3}{20}$
2. $\frac{4}{9}\times\frac{3}{8}$
3. $\frac{25}{3}\times2\frac{1}{5}$
4. $14\frac{1}{3}\times9$
5. $\frac{3}{10}\times\frac{5}{6}\times\frac{4}{9}$
6. $1\frac{2}{3}\times2\frac{1}{4}\times3\frac{1}{5}$

Divide:

7. $\frac{3}{8} \div \frac{3}{2}$
8. $\frac{3}{4} \div \frac{7}{8}$
9. $10\frac{1}{2} \div \frac{3}{4}$
10. $39 \div 4\frac{1}{3}$

Practical Use of Fractions

The practical uses of fractions are limited in everyday experience very largely to halves, fourths, eighths, sixteenths and thirty-seconds, as used by carpenters, mechanics, and machinists; to these may be added tenths, used by architects and draftsmen. The shopkeeper frequently uses (in addition to halves and fourths) thirds, sixths and twelfths to designate fractional parts of a dozen. For household purposes, halves, fourths and eighths usually suffice, whether dealing with inches, feet and yards, or pounds and ounces, or tablespoons and cups, quarts and pints. Fractions like $\frac{5}{7}$ or $\frac{2}{9}$ or $\frac{3}{11}$ rarely occur in ordinary, everyday affairs; once in a while fifths are encountered, but these are generally expressed as tenths, or decimals. We may expect, then, to have to use fractions in carpenter work, in the shop, in the store, when marketing, when cooking, or when sewing. Fractions are apt to arise whenever we *measure* something.

EXAMPLES

1. A mechanic receives 72¢ per hour. If he works $38\frac{1}{4}$ hours per week, what are his weekly wages?

$$38\frac{1}{4} \times 72¢ = \frac{153}{\cancel{4}} \times \overset{\$.18}{\cancel{72}} = \$27.54, \textit{ Ans.}$$

2. What is the cost of $4\frac{3}{4}$ lb. of meat at $28\frac{1}{2}$¢ per lb.?

$$4\frac{3}{4} \times 28\frac{1}{2}¢ = \frac{19}{4} \times \frac{57}{2} = \frac{1083}{8} = \$1.35, \textit{ Ans.}$$

PROBLEMS FOR PRACTICE: NO. 17

1. A salesgirl in a dry goods store sold $2\frac{1}{2}$ yd. of ribbon to one customer and $3\frac{3}{4}$ yd. to another. If the spool from which she cut these pieces originally contained $22\frac{1}{2}$ yd., how many yards remained?

2. Mr. Cushman's car averages $17\frac{1}{2}$ miles on a gallon of gasoline. How many gallons will he need for a trip of 350 miles? How much will the gasoline for the trip cost at $18\frac{1}{2}$¢ per gallon?
3. According to a certain recipe $3\frac{1}{2}$ cups of sugar and $2\frac{1}{4}$ cups of milk are required for 12 portions. How many cups of each will Mrs. Jones need if she wishes to make enough for 8 portions?
4. If ice is approximately $\frac{9}{10}$ as heavy as water, and water weighs $62\frac{1}{2}$ lb. per cubic foot, what is the weight of a block of ice occupying 4 cubic feet?
5. For working $6\frac{1}{4}$ hours overtime, a man was paid at the rate of $1\frac{1}{2}$ times the regular wage rate. If his regular hourly rate was 64¢ an hour, how much did he receive?

DECIMALS

Decimal Fractions

Fractions with a denominator of 10, 100, 1000, etc. may be written as *decimals,* i.e., without fraction line and without expressing the denominator in numbers; thus .8 is the same as $\frac{8}{10}$, 0.24 is the same as $\frac{24}{100}$, and .352 is the same as $\frac{352}{1000}$. In all decimal fractions the denominators are multiples of 10. They do not have to be written out, since the position of the decimal point takes the place of the denominator.

Adding and Subtracting Numbers with Decimals

These operations are performed exactly as with ordinary numbers; the important thing to remember is that the decimal points must be kept one under the other, and that the decimal point in the answer must be placed directly beneath the other decimal points.

Multiplying Decimals

When decimal numbers are multiplied, the operation is carried out exactly as in the cast of non-decimal numbers, except that, in the product, the decimal point is "pointed off" as many places, beginning at the right, as there are in the multiplier and the multiplicand together. For example:

(a)	(b)
28.63	37.136
×.04	2.05
1.1452	185680
	74272
	76.12880

Checking by Estimating

In order to prevent careless mistakes in pointing off properly in the product, it is always helpful to estimate the result roughly before actually multiplying; thus in finding the product of 4.876 by 297.8, we should expect the result to be in the neighborhood of 5×300, or 1500. Hence when the product is found, we know that 1452.0728 is correct, and that 145.20728 or 14,520.728 is incorrect.

Changing a Common Fraction to a Decimal

Any common fraction, proper or improper, may be changed to a decimal fraction by simply dividing, and carrying the division to as many decimal places as desired.

EXAMPLE

Change $\frac{8}{23}$ to a decimal fraction, (a) expressing the result to the nearest tenth; (b) to the nearest hundredth; (c) to the nearest thousandth.

```
       .347
 23 )8.000
      6 9
      1 10
        92
       180
       161
        19
```

Ans.:

(a) .3
(b) .35
(c) .348

Dividing Decimals

When dividing by a decimal, the decimal point in both the divisor and the dividend must be moved as many places to the right as there are decimal places in the divisor.

EXAMPLE

Divide 86.345 by 7.23.

```
          11.94
723 )8634.50
      723
      ----
      1404
       723
       ----
       681 5
       650 7
       -----
        30 80
        28 92
        -----
```

By the Hundred; by the Thousand

Insurance premiums, taxes, prices of goods, and so on are frequently expressed as so much "per hundred" or so much "per thousand." In the first case, if the unit is pounds, the price may be given as \$15 per cwt. (hundredweight); in the latter case, it is sometimes written as \$8 per M. To find the cost in these cases, reduce the given quantity to hundreds (or thousands) by dividing by 100 (or 1000), simply by pointing off the appropriate number of decimal places, and then multiply by the quoted price per 100 (or per 1000).

EXAMPLE

Find the cost of 5400 tiles at \$22.50 per thousand.

$5400 \div 1000 = 5.4$

$\$22.50 \times 5.4 = \$121.50$, *Ans.*

PERCENTAGE

So Many per Hundred

A per cent is just another way of expressing a fractional part of a quantity, only the basis of the comparison is so many parts out of every hundred. That is, the denominator is always 100 in the case of a per cent. Moreover, since the denominator is not written, but understood, a per cent does not look like a fraction; the per cent sign (%) takes the place of the fraction line and the denominator. But it tells the same story, and, as we shall see, is usually much more convenient than the fraction.

$$8\% = \frac{8}{100} = .08 = \frac{2}{25};$$
$$10\% = \frac{10}{100} = .10 = \frac{1}{10};$$
$$40\% = \frac{40}{100} = .4 = \frac{2}{5};$$
$$3\tfrac{1}{2}\% = \frac{3.5}{100} = .035 = \frac{7}{200}; \text{ etc.}$$

TABLE OF COMMON FRACTIONS AND EQUIVALENT PER CENTS

$\frac{1}{2} = 50\% = .5$	$\frac{1}{5} = 20\% = .2$	$\frac{5}{6} = 83\frac{1}{3}\% = .833+$
$\frac{1}{4} = 25\% = .25$	$\frac{2}{5} = 40\% = .4$	$\frac{1}{8} = 12\frac{1}{2}\% = .125$
$\frac{3}{4} = 75\% = .75$	$\frac{3}{5} = 60\% = .6$	$\frac{3}{8} = 37\frac{1}{2}\% = .375$
$\frac{1}{3} = 33\frac{1}{3}\% = .33+$	$\frac{4}{5} = 80\% = .8$	$\frac{5}{8} = 62\frac{1}{2}\% = .625$
$\frac{2}{3} = 66\frac{2}{3}\% = .66+$	$\frac{1}{6} = 16\frac{2}{3}\% = .163+$	$\frac{7}{8} = 87\frac{1}{2}\% = .875$

Finding a Per Cent of a Number

This is perhaps the most frequent use made of percentage in practical affairs. It is simply equivalent to multiplying the number by the given per cent expressed as a decimal. The number of which we wish to find a certain per cent is called the *base;* the per cent to be taken is called the *rate;* and the amount so found is called the *percentage.*

EXAMPLES

1. Find 35% of $44.56.

$$
\begin{array}{rl}
\$44.56 & \text{(base)} \\
.35 & \text{(rate)} \\
\hline
22280 & \\
13368 & \\
\hline
\$15.596, \text{ or } \$15.60 & \text{(percentage)}
\end{array}
$$

2. What is 4½% of $138?

$$
\begin{array}{rl}
\$138 & \text{(base)} \\
.045 & \text{(rate)} \\
\hline
690 & \\
552 & \\
\hline
\$6.21 & \text{(percentage)}
\end{array}
$$

RULE: To find a given per cent of a given number, multiply the number (base) by the rate (expressed as a decimal).

Finding What Per Cent One Number Is of Another

This is a convenient and useful way of comparing one quantity with another. It is constantly used in business, in science, and in technical work. It is equivalent to finding the rate when we know both the percentage and the base.

EXAMPLES

1. What per cent of $256 is $16?

$$\frac{\$\ 16}{\$256} = \frac{1}{16} = .0625 = 6\tfrac{1}{4}\%$$

2. What per cent of 150 lb. is 80 lb?

$$\frac{80}{150} = \frac{8}{15} = .533 = 53.3\%$$

3. A coat is reduced from $22.50 to $18. What is the per cent of reduction?

$22.50—$18=$4.50, the amount of reduction

$$\frac{\$\ 4.50}{\$22.50}=\frac{45}{225}=\frac{3}{15}=\frac{1}{5}=20\%$$

RULE: To find the rate per cent, divide the percentage by the base and reduce the resulting fraction to a decimal; express the decimal as a per cent.

Finding a Number When a Certain Per Cent of It Is Given

It sometimes happens that we know a certain per cent of something, and we wish to determine what that entire "something" is. This is equivalent to the question: Given the percentage as well as the rate, what is the base?

EXAMPLES

1. $36 is 60% of what amount?

$$60\%=\frac{6}{10}=\frac{3}{5}$$
$$\$36\div\frac{3}{5}=\$36\times\frac{5}{3}=\$60,\ \textit{Ans.}$$
$$\text{or, } \frac{\$36}{.6}=\frac{\$360}{6}=\$60$$

2. If $72 represents 16% of the selling price of an oil burner, what is the entire selling price?

$$\$72\div.16=\$450,\ \textit{Ans.}$$

3. What is the original amount of a bill if a 2% discount for cash comes to $1.73?

$$\$1.73\div.02=\$86.50,\ \textit{Ans.}$$

RULE: To find the base (entire amount), divide the percentage by the rate expressed either as a common fraction or as a decimal.

PROBLEMS FOR PRACTICE: NO. 18

1. Find 3.16% of \$2400.
2. What per cent of \$56 is \$12?
3. Express $9/32$ as a per cent.
4. \$136 is what per cent of \$400?
5. What is $1\frac{1}{2}$% of \$6875?
6. 4.8 lb. is what per cent of 24 lb.?
7. A tax of $4\frac{1}{2}$¢ a gallon on gasoline, included in a total cost of 18¢ a gallon, is a tax of what per cent?
8. A salesman receives a commission of $8\frac{1}{2}$% on sales amounting to \$3864 worth of merchandise. What is the amount of his commission?
9. Thirty-five candidates failed in an examination. If 14% failed, how many candidates were there?
10. A man pays \$187.50 interest a year on his mortgage. If the rate is 5%, what is the amount of the mortgage?

Further Use of the Idea of "Per Hundred"

In business and financial matters, per cents less than 1%, or fractional parts of 1%, are frequently used. These may be written in several ways; e.g., $\frac{1}{2}$ of $1\%=\frac{1}{2}\%=.5\%=.005$; or, $\frac{1}{8}$ of $1\%=\frac{1}{8}\%=.12\frac{1}{2}\%=.00125$; etc. Another device sometimes employed is the *mill;* one mill$=\frac{1}{10}$¢$=\$.001$. Thus a tax rate of "2.5 mills on the dollar" means a tax of \$.0025, or $\frac{1}{4}$¢, on every dollar; this is equivalent to 25¢ on every \$100, or \$2.50 on every \$1000. This idea of so much per hundred is also commonly used in a slightly different way, as, for example, when an insurance premium is quoted as "38¢ per \$100"; or a brokerage fee as "\$17.50 per 100 shares"; or a bankruptcy settlement as "74¢ on the dollar."

EXAMPLES

1. If the tax on a house is increased by $\frac{3}{10}$%, what is the amount of the increase on a house valued at \$13,850?

$$\$13{,}850\times.003=\$41.55,\ \textit{Ans.}$$

2. A bankrupt firm pays 48¢ on the dollar. How much will a creditor receive whose claim amounts to $132.50?

$$\$132.50 \times .48 = \$63.60, \textit{ Ans.}$$

3. An insurance premium is quoted at "14¢ a hundred dollars." What is the amount of the premium on a policy of $8400?

$$84 \times \$.14 = \$11.76, \textit{ Ans.}$$

4. The tax on property valued at $22,500 is 21.4 mills. What is the amount of the tax?

$$225 \times \$2.14 = \$481.50, \textit{ Ans.}$$

PROBLEMS FOR PRACTICE: NO. 19

1. Depreciation on a piece of property is figured at ½ of 1% a year. If the value of the property is $250,000, what is the amount of annual depreciation?
2. The sales tax on an automobile is 3½%. If the car costs $875, what is the amount of the tax?
3. In a town where the tax rate is 12.31 mills, what is the tax on a house worth $8400?
4. A fire insurance policy for $2500 costs 8½¢ per $100. What is the cost of the policy?
5. A tax fine of ⅛% on $30,000 amounts to how much?

Per Cent of Increase or Decrease

Another common and useful application of percentage is to describe the amount of an increase or a decrease as a per cent. We say, for example, that rentals have increased 5%, or that the automobile accident rate has dropped 8%, or that the dollar will buy 4% less, and so on.

EXAMPLES

1. A table lamp originally sells for $10.75. If the price is reduced by 20% at a sale, what is the sale price?

$$\$10.75 \times .2 = \$2.15$$
$$\$10.75 - \$2.15 = \$8.60, \textit{ Ans.}$$

2. Mr. Bogart's salary is increased by 10%. If he formerly received \$36 per week, what is his new salary?

$$\$36 \times .1 = \$3.60$$
$$\$36 + \$3.60 = \$39.60, \textit{ Ans.}$$

3. If the average daily sales of a small shop increased from \$200 to \$240 per day, what per cent of increase is this?

$$\$240 - \$200 = \$40$$
$$\$40 \div \$200 = \tfrac{4}{20} = 20\%, \textit{ Ans.}$$

4. A new automobile costing \$995 is worth \$597 one year later. By what per cent has it decreased in value during its first year?

$$\$995 - \$597 = \$398$$
$$\$398 \div \$995 = .4 = 40\%, \textit{ Ans.}$$

5. A bank drops its interest rate from 3½% to 3%. What is the per cent of decrease in the interest rate?

$$3\tfrac{1}{2}\% - 3\% = \tfrac{1}{2}\%$$
$$\frac{\tfrac{1}{2}\%}{3\tfrac{1}{2}\%} = \tfrac{1}{2} \times \tfrac{2}{7} = \tfrac{1}{7} = 14\tfrac{2}{7}\%, \textit{ Ans.}$$

Finding the Base When the Rate Per Cent of Increase or Decrease Is Known

Occasionally, instead of the actual amount of an increase or decrease, we are given the rate per cent of the increase. In other words, we know the rate of increase, and the increased amount, and we wish to find the original amount.

EXAMPLES

1. This year Mr. Carter's income tax amounts to \$93, which is 20% more than he paid last year. How much did he pay last year?

$$100\% = \text{base, last year's tax}$$
$$100\% + 20\% = 120\% = \text{this year's tax}$$
$$120\% = \$93$$
$$1\% = \frac{1}{120} \times \$93 = \$.775$$
$$100\% = 100 \times \$.775 = \$77.50, \textit{ Ans.}$$

or, $100\%+20\%=120\%=1.2$
$\$93\div1.2=\$77.50$

2. At a bargain sale Mrs. Wexler bought a dress at 25% less than the original price. If she actually paid \$13.50 for the dress, what was the original price?

$100\%=$base, original price
$100\%-25\%=75\%$, sale price
$75\%=\$13.50$

$$1\%=\frac{1}{75}\times\$13.50=\$.18$$

$100\%=100\times\$.18=\$18$, *Ans.*
or, $\$13.50\div.75=\$18$

PROBLEMS FOR PRACTICE: NO. 20

1. \$6000 is 120% of what amount?
2. \$360 is 40% of what amount?
3. \$1260 is 70% less than what amount?
4. \$920 is 15% more than what amount?
5. In a certain recent year a merchant's sales showed an increase of 16% over his last year's sales, which were \$24,600. What were his sales this year?
6. A certain brand of marmalade, according to the label, contains $\frac{2}{10}$ of 1% of preservative. How many ounces of this preservative are contained in a 13½ oz. jar of marmalade when full?
7. An increase of 4¢ per pound in the price of meat that has been selling for 29¢ a lb. is a percentage increase of how much?
8. A dealer purchased a cabinet radio for \$338, which was 35% less than the list price. What was the list price?
9. By increasing their sales quota 22%, a corporation's sales quota amounted to \$54,900. What was the previous quota?
10. A shopkeeper sells his merchandise at a net profit of 16% of his sales. What volume of sales are required to yield a net profit of \$5600?

RATIO AND PROPORTION

Meaning of Ratio

A ratio is a device for comparing two quantities of the same sort. For example, if two boards are 8 ft. and 10 ft. long, respectively, we could say that the second is 2 ft. longer than the first, or 25% longer. This is a *difference* method of comparing them; or, telling how much more or less. Another way of comparing them would be to say that one is $\frac{4}{5}$ as long as the other, or the second is $\frac{5}{4}$, i.e., $1\frac{1}{4}$ times as long as the first. This is the *ratio* method of comparison. We say that the lengths of the boards are "in the ratio of 4 to 5, or 5 to 4," which may be written 4 : 5, or 5 : 4. A ratio, then, is simply a *fraction* which tells the comparison at a glance. The above ratio could just as well be written as $\frac{4}{5}$ or $\frac{5}{4}$; in fact, the colon (:) is really an abbreviation for "÷" with the horizontal line omitted. Notice that a ratio is independent of the units of measure; i.e., the two boards mentioned above are in the ratio of 4 : 5 whether we express their lengths as inches, feet, or yards. The units "cancel out," and the ratio remains 4 : 5.

EXAMPLES

1. A photographic plate measures $4\frac{1}{4}''$ by $5\frac{1}{2}''$. What is the ratio of its dimensions?

   $4\frac{1}{4}=\frac{17}{4}$ $\qquad$ $5\frac{1}{2}=\frac{11}{2}$

   $\frac{17}{4}\div\frac{11}{2}=\frac{17}{4}\times\frac{2}{11}=\frac{17}{22}$, or 17 : 22, *Ans.*

2. A pictorial representation of an animal in a textbook is labeled as "$\frac{2}{5}$ actual size." If the length of the animal in the drawing is 3.8 cm., what is its actual length?

   $3.8\times\frac{5}{2}=9.5$ cm., *Ans.*

3. The working model of a machine is to be on a scale of 1 : 50. If a connecting piece of this machine is actually 16′8″ long, how long should the corresponding piece of the model be made?

   $16'8''=200''$ $\qquad$ $200''\times\frac{1}{50}=4''$, *Ans.*

Ratio and Per Cent

Since a ratio is always a fraction, ratios are frequently expressed as per cents. The specific gravity of a substance is the ratio of its weight to the weight of an equal volume of water. Thus ether, being only about 70% as heavy as water, has a specific gravity of 0.715; ice, 0.917; air, 0.0013; aluminum, 2.6; lead, 11.37; etc.

EXAMPLES

1. If a bankrupt firm can pay 55¢ on the dollar, that means that its ratio of assets to liabilities is 0.55. If its assets amount to $22,500, what are its liabilities?

   $\$22{,}500 \div .55 = \$40{,}909.09$, *Ans.*

2. The financial record of a utility company shows that "its interest requirements were earned 3.8 times." If the interest requirement is $250,000, what were the earnings that year?

   $\$250{,}000 \times 3.8 = \$950{,}000$, *Ans.*

3. If a family budgets its income so as to allow 22% for rent and 14% for clothing, (a) what is the ratio of the cost of clothing to the cost of shelter; (b) how much can be spent for rent per month on an annual income of $2400?

   (a) $14\% \div 22\% = \frac{14}{22} = \frac{7}{11}$, or 7:11, *Ans.*

   (b) $\frac{\$2400}{12} \times .22 = \$44$, *Ans.*

4. A mixture of concrete is made up of 1 part of cement, 2½ parts of sand, and 4 parts of stone. Find (a) the ratio of sand to stone; (b) the ratio of cement to sand; (c) what per cent of the concrete mixture is sand?

   (a) $2\frac{1}{2} \div 4 = \frac{5}{2} \times \frac{1}{4} = \frac{5}{8}$, or 5:8, *Ans.*
   (b) $1 \div 2\frac{1}{2} = 1 \times \frac{2}{5} = \frac{2}{5}$, or 2:5, *Ans.*
   (c) $2\frac{1}{2} \div 7\frac{1}{2} = \frac{5}{2} \times \frac{2}{15} = \frac{1}{3}$, or $33\frac{1}{3}\%$, *Ans.*

Scale Drawings

In representing distances on a map, or dimensions on a plan or blueprint, it is necessary to use a scale, or to "scale down" the quantities, all in the same ratio. Thus on a given map, an inch might represent 500 miles, in which case two cities located $1\frac{1}{2}$ inches apart on the map would actually be 750 miles distant from each other. Or the floor plan of a house might be drawn to a scale of $1''=10$ ft.; in that case a room which on the plan is $1\frac{1}{2}$ inches wide is actually 15 feet wide, and a room 24 feet long would be represented by a line 2.4 inches long.

EXAMPLES

1.

	Scale used	*Actual length*	*Scale length*
(a)	$1''=20$ ft.	85 ft.	$4\frac{1}{4}$ in.
(b)	$\frac{1}{8}''=1$ ft.	44 ft.	$5\frac{1}{2}$ in.
(c)	$\frac{1}{4}''=10$ ft.	250 ft.	$6\frac{1}{4}$ in.
(d)	$1''=1$ ft.	$14\frac{1}{2}$ ft.	$14\frac{1}{2}$ in.
(e)	$1''=50$ mi.	240 mi.	4.8 in.

2. The blueprint of a house shows a bedroom $3\frac{1}{2}''$ by $6\frac{1}{4}''$. If the scale of the blueprint is $\frac{1}{4}''=1'$, what are the actual dimensions of the room?

$\frac{7}{2}\div\frac{1}{4}=\frac{7}{2}\times4=14$ ft., width;
$\frac{25}{4}\div\frac{1}{4}=\frac{25}{4}\times4=25$ ft., length.

3. A map is drawn to a scale of $1''=75$ mi. How far apart on this map are two cities which are actually 450 miles apart?

$450\div75=6$ in. apart.

Meaning of Proportion

The word "proportion" is often used carelessly, or with only a vague notion of what is really meant. Strictly speaking, when we compare only *two* quantities, we cannot speak of their "proportion"; we can refer to the ratio between them, or what part or what per cent one is of the other. When we speak of a proportion, we have in mind *four* quantities, and are comparing

them *in pairs,* as ratios. In other words, if two ratios are equal to each other, they are said to form a proportion. Thus, $3:5=12:20$ is a proportion. Putting it another way, if the ratio between any two quantities is equal, numerically, to the ratio between two other quantities, then the four quantities are in proportion. For example, a nickel bears the same ratio to a dime that a half-dollar does to a dollar, since

$$5/10=50/100, \text{ or } 1/2=1/2.$$

Notice that in the first case all four quantities are expressed in terms of the same units, viz., cents, although the units don't appear; in the second case, the two ratios have been reduced to lowest terms to show their equality. Or again, if a man 6 ft. tall casts a shadow 8 ft., then a pole 18 ft. high will cast a shadow 24 ft. long; or $6:8=18:24$;

$$6/8=18/24, \text{ or } 3/4=3/4.$$

EXAMPLES

1. If $2/3=8/n$, find the missing number n.

Multiplying "crisscross":

$$2\times n=8\times 3$$
$$2n=24 \qquad\qquad \text{or, } 2/3=8/12$$
$$n=12$$

2. If $4/5=n/20$, find the missing quantity n.

Cross-multiplying:

$$4\times 20=5\times n$$
$$5n=80 \qquad\qquad \text{or, } 4/5=16/20$$
$$n=16$$

3. If $3/n=5/8$, find n.

Multiplying: $3\times 8=5\times n$

$$5n=24$$
$$n=4\tfrac{4}{5} \qquad\qquad \text{or, } \frac{3}{4\frac{4}{5}}=5/8$$

4. If $n/4 = 2/7$, find n.

$$7 \times n = 2 \times 4$$
$$7n = 8$$
$$n = 8/7 \qquad \text{or,} \quad \frac{1\frac{1}{7}}{4} = 2/7$$

PROBLEMS FOR PRACTICE: NO. 21

1. If a photograph is to be enlarged to a width of 14″, and the original negative measures 3½″ wide by 4½″ long, what will the length of the enlargement be?
2. The diagram of an article in a catalog is described as being "¾ actual size." If the diagram measures 2½″×5¼″, what are the actual dimensions of the article?
3. A clerk's salary is increased from \$25 to \$28 per week. (a) What per cent of increase is this? (b) What is the ratio of his old salary rate to his new salary rate?
4. The Anchor Textile Corp., declaring bankruptcy, pays 64¢ on the dollar. What amount can they pay to a creditor to whom they owe a bill of \$315?
5. The operating expense ratio in a certain business is .185, which means that 18½% of the total sales volume is allowed for operating expenses. If their average monthly expenses are \$1480, what should the monthly sales volume be?
6. A chemical mixture consists of 4 parts of ingredient A, 3½ parts of ingredient B, and 1½ parts of ingredient C. (a) What per cent of the mixture consists of ingredient C? (b) What is the ratio of ingredient B to C?
7. The working drawing of a wooden cabinet calls for a piece of wood 27″ long. If the scale of the drawing is 1″=1′, how long should this piece appear on the drawing?
8. What is the distance in feet between two places on a map drawn to a scale of 1 : 500, if the two points are 7½ inches apart on the map?

CHAPTER III

WEIGHTS AND MEASURES

No sooner had primitive man learned to count than he very probably began to measure. His practical needs for measuring were doubtless plentiful.

Man's earliest units were suggested by everyday activities: a stone's throw, a day's journey, or a *furlong* as in plowing a furrow in a field. Smaller units of length were suggested by various parts of the body; e.g., the *foot* which varied from about 10″ to 19″; the *yard,* which was the length of the outstretched arm from shoulder to finger tip; the *inch,* which at first was probably the length of the thumb joint, but later became the length of three grains of barley. Quite common at one time were the *span,* or the spread of the hand, about 9″; the *palm,* or width across an open hand at the base of the fingers, about 3″; the *cubit,* the length of the forearm from the elbow to the end of the middle finger, about 20″; and the *digit,* or breadth of the finger, about $\frac{3}{4}$″. In the 16th century the *rod* was defined as "the length of the left foot of 16 men lined up as they left church on Sunday morning." The *mile* has come down to us as the distance covered by a man walking 1000 (mille) paces, which, as might be expected, originally varied from $\frac{5}{8}$ to $\frac{6}{7}$ of a mile, since a pace varied anywhere from $2\frac{1}{2}$ to 5 feet.

LINEAR MEASURE

Units of Length

Today the commonly used English units of length and distance are limited to the following:

LINEAR MEASURE

12 inches (in.)	= 1 foot (ft.)	320 rods	= 1 mile (mi.)
3 feet	= 1 yard (yd.)	1760 yards	= 1 mile
5½ yards = 16½ feet	= 1 rod (rd.)	5280 feet	≐ 1 mile

EXAMPLES

1. A swimming pool is 12¾ yd. wide; how many feet and inches is this?

$$12\tfrac{3}{4} \times 3 = 38\tfrac{1}{4} \text{ ft.} = 38 \text{ ft. } 3 \text{ in., } \textit{Ans.}$$

2. A fence is 250 ft. long; how many yards of wire screening are required to cover this fence?

$$250 \div 3 = 83\tfrac{1}{3} \text{ yd., or } 83 \text{ yd. } 1 \text{ ft., } \textit{Ans.}$$

3. Find the sum of 3 yd. 2 ft. 8 in., 4 yd. 5 ft. 9 in., and 16 ft. 5 in.

	3 yd.	2 ft.	8 in.
	4 yd.	5 ft.	9 in.
		16 ft.	5 in.
	7 yd.	23 ft.	22 in.
or	7 yd.	24 ft.	10 in.
or	15 yd.	10 in., *Ans.*	

Perimeter

The total distance around a plane figure is called its *perimeter.* Thus the perimeter of a square is the sum of its four sides, or four times the length of any one of its sides. The perimeter of a rectangle equals twice its length plus twice its width, or twice the sum of its length and width.

$$P \text{ of square} = 4s$$
$$P \text{ of rectangle} = 2l + 2w = 2(l + w)$$

EXAMPLES

1. How many yards of trim are needed for a rectangular piece of fabric 45″ long and 18″ wide?

$$P=2(45+18)=126 \text{ in.}$$

$$126 \text{ in.}=\frac{126}{36}=3^{18}/_{36}=3\tfrac{1}{2} \text{ yd., } \textit{Ans.}$$

2. A city block in the shape of a rectangle is three times as long as it is wide. If its perimeter is 960 ft., what are its dimensions?

$$l=3w$$
$$P=2(l+w)$$
$$\text{or } P=2(3w+w)=2(4w)=8w$$
$$\text{i.e., } 8w=960$$
$$960\div8=120, \text{ width}$$
$$3\times120=360, \text{ length, } \textit{Ans.}$$

Metric Units of Length

By the 18th century there had grown up in Europe a confusing variety of units and standards of weights and measures of all sorts. During the chaos of the French Revolution the opportunity was seized to establish a more uniform system of measures, the *metric* system, which was adopted in France in 1799, and is now used in nearly every civilized country in the world except the United States and Great Britain. Curiously enough, before its inception in France, a somewhat similar system had been suggested in America by Thomas Jefferson.

The fundamental standard of length in the metric system is the *meter,* defined as the distance between two scratch-marks on a particular bar of platinum-iridium (carefully preserved at the International Bureau of Weights and Measures, near Paris) when the temperature of the bar is that of melting ice (0° Centigrade). This particular alloy is used because it is scarcely subject to changes in length as the temperature varies, and because it will not tarnish or oxidize. All other units of length are multiples or sub-multiples of the meter, as shown.

METRIC LINEAR MEASURE

10 millimeters (mm.)	= 1 centimeter (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 meter (m.)
10 meters	= 1 decameter (Dm.)
10 decameters	= 1 hectometer (Hm.)
10 hectometers	= 1 kilometer (Km.)
10 kilometers	= 1 myriameter (Mm.)

It should be noted, however, that practically the only ones that are commonly used are the kilometer, the meter, the centimeter and the millimeter. For convenience in converting measures from either system to the other, the following approximate equivalents are helpful:

APPROXIMATE EQUIVALENTS

1 centimeter	= about 0.4 inch
1 meter	= about 1.1 yard
1 kilometer	= about 0.6 mile
1 inch	= about 2.5 centimeters
1 yard	= about 0.9 meter
1 mile	= about 1.6 kilometers

EXAMPLES

1. The distance between two European cities is given in a newspaper account as 850 kilometers. How many miles is this?

   $850 \times 0.6 = 510$ miles, *Ans.*

2. A photographic film size as stated on the wrapper is 6.5 cm. $\times$ 11 cm. What are these dimensions in inches?

   $6.5 \times 0.4 = 2.6$ in.
   $11 \times 0.4 = 4.4$ in. *Ans.*

   Note: Using 1 in.=2.54 cm., these dimensions are 2.56 in. $\times$ 4.33 in.; on the wrapper they are given as $2\frac{1}{2}'' \times 4\frac{1}{4}''$.

3. At a track meet a runner covered the 1000-meter course in 2:32:4, or 2 minutes and 32⅘ seconds. What was his average speed in feet per second?

1 meter=1.1 yd.=3.3 ft., approximately
2 min. 32⅘ sec.=153 sec., approximately

$$\frac{1000\times 3.3}{153}=21.6 \text{ ft. per sec., } \textit{Ans.}$$

SURFACE MEASURE

Units of Area

The area or surface measure of a figure is measured in "square units," derived from corresponding linear units. The most frequently encountered units of area are:

UNITS OF SURFACE MEASURE

144	square inches (sq. in.)	=	1 square foot (sq. ft.)
9	square feet	=	1 square yard (sq. yd.)
30¼	square yards	=	1 square rod (sq. rd.)
160	square rods	=	1 acre (A.)
640	acres	=	1 square mile (sq. mi.)

Area of a Rectangle

The surface included by a rectangle is easily seen to be the product of its length by its width. In the special case of a square, where the length equals the width, the area is found by multiplying the length of a side by itself (called squaring a number).

$$A \text{ of square} = s^2$$
$$A \text{ of rectangle} = l \times w$$

The areas of other geometric figures, such as triangles, circles, cubes, rectangular solids, cylinders and the like, are discussed in Chapter V.

EXAMPLES

1. The dimensions of a cellar floor are 32 ft.×48 ft. How many sq. ft. of floor surface are there? If a gallon of deck paint covers 160 sq. ft., how many gallons of paint are required for this floor?

$48\times 32=1536$ sq. ft., *Ans.*
$1536\div 160=9.6$ gallons, *Ans.*

2. The rectangular floor of a playroom, 18′×24′, is to be covered with linoleum at 9½¢ a sq. yd. What will be the cost?

$$\frac{18\times 24}{9}\times\frac{\$.19}{2}=\$4.56,\ \textit{Ans.}$$

3. A typical city block is 200 ft. wide and 480 ft. long. How does this compare in area with an acre of land?

$$\frac{5280\times 5280}{640}=43{,}560,\ \text{number of sq. ft. in one acre.}$$

$$\frac{43{,}560}{200\times 480}=.45+,\ \text{or } 45\%.$$

hence an acre is about ½ of a city block, or, a typical city block is approximately equal to a little more than 2 acres in area. *Ans.*

Metric Surface Measure

The basic unit of metric surface measure is the *square meter,* which is a square one meter long on a side. Another convenient and commonly used unit of area is the square centimeter, which is a square one centimeter on each side. The table that follows shows the relation between the various units of area.

METRIC SURFACE MEASURE

1 square millimeter (sq. mm.)	=	.000001 square meter
1 square centimeter (sq. cm.)	=	.0001 square meter
1 square decimeter (sq. dm.)	=	.01 square meter
1 square kilometer (sq. Km.)	=	1,000,000 square meters

For ordinary purposes and convenience in converting from English into metric units and vice versa, the following equivalents are given.

APPROXIMATE EQUIVALENTS

1 sq. inch	= about	6.5	sq. centimeters
1 sq. foot	= about	.09	sq. meter
1 sq. yard	= about	.84	sq. meter
1 sq. mile	= about	2.6	sq. kilometers
1 sq. centimeter	= about	.15	sq. inch
1 sq. kilometer	= about	.39	sq. mi.

EXAMPLES

1. The Library of Congress card for a book states that the pages of the book are 13 cm. $\times$ 21.5 cm. How many square inches are there to a page?

$$\frac{13\times 21.5}{6.5}=43 \text{ sq. in., } \textit{Ans.}$$

2. A public park occupies .75 of a square kilometer. How many acres is this?

$$.75\times .39\times 640=187+ \text{ acres, } \textit{Ans.}$$

CUBIC CONTENTS

Measure of Volume

The terms *volume* and *cubic contents* mean essentially the same thing, viz., the amount of space an object takes up, or its bulk. In the case of a hollow solid, such as a rectangular box, its *capacity* is exactly the same as its volume if the thickness of the sides, top and bottom are disregarded, or if "inside" measurements are taken. To find the capacity of a storage bin or a cylindrical tank, therefore, we simply find its volume from its inside dimensions and call that its capacity, or cubic contents.

Volume and cubic contents are measured in units derived from linear units, the commonest being the cubic inch, the cubic foot and the cubic yard.

UNITS OF VOLUME

1728 cubic inches (cu. in.)	=	1 cubic foot (cu. ft.)
27 cubic feet	=	1 cubic yard (cu. yd.)
128 cubic feet	=	1 cord (cd.)

The volume or bulk of some commonly used materials is measured in special units. Thus earth and sand are measured in loads, a *load* being equal to 1 cu. yd. Wood is sometimes measured in cords, a *cord foot* being a pile of wood 4 ft. $\times$ 4 ft. $\times$ 1 ft. In measuring stone, a perch is used, a *perch* being a pile 16½ ft. $\times$ 1½ ft. $\times$ 1 ft., and containing on the average 24¾ cu. ft.

Other Measures of Capacity

The capacity of variously shaped containers, both for liquids as well as solids, whether bottles or barrels, cans or crates, is not generally measured in cubic inches or cubic feet, but in pints, quarts, pecks, bushels, and so on. The conventional measures used in the United States today are:

MEASURES OF CAPACITY

Liquid Measure			Dry Measure		
4 gills (gi.)	=	1 pint (pt.)	2 pints	=	1 quart (qt.)
2 pints	=	1 quart (qt.)	8 quarts	=	1 peck (pk.)
4 quarts	=	1 gallon (gal.)	4 pecks	=	1 bushel (bu.)

Metric Measures of Capacity

In measuring cubical contents by the metric system the fundamental unit of volume is the *cubic meter*. Other units commonly employed follow:

METRIC UNITS OF VOLUME AND CAPACITY

1 cubic millimeter (cu. mm.)	=	.000000001 cubic meter
1 cubic centimeter (cu. cm.)	=	.000001 cubic meter
1 cubic decimeter (cu. dm.)	=	.001 cubic meter
1 centiliter (cl.)	=	.01 liter
1 deciliter (dl.)	=	.1 liter
1 hektoliter (Hl.)	=	100 liters

For measuring both liquids and solids, the *liter* is commonly employed. A liter is the same as a cubic decimeter, or 1000 cu. cm. (also abbreviated *cc.*).

APPROXIMATE EQUIVALENTS

1 cubic inch	=	about 16 cubic centimeters
1 cubic centimeter	=	about .06 cubic inch
1 quart (liquid)	=	about .95 liter
1 liter	=	about 1.06 quart (liquid)

EXAMPLES

1. How many cubic inches are occupied by a liter? a liquid quart?

   1000×.06=60 cu. in. to 1 liter
   .95×60=57 cu. in. to 1 quart

2. How many cc. are there in ½ pint of cream?

   ½ pt.=.25 qt.
   .25×.95×1000=237 cc., *Ans.*

3. A physician's prescription calls for 100 cc. of medicine. How large a bottle will the druggist need, if his bottles are in ounce-sizes?

   1 qt.=32 (fluid) oz.
   100 cc.=0.1 liter=.106 qt.

   32×.106=3.4 oz., or a 4-oz. bottle, *Ans.*

MEASURES OF WEIGHT

English System of Weights

Several systems of English weights are in use today: (1) *Avoirdupois;* (2) *Troy;* and (3) *Apothecaries'*. They are given here for reference. The first is used for all ordinary measurements, and is to be understood if not otherwise specified; the second is used by jewelers in weighing gold, silver, platinum, and precious stones; the third is used by pharmacists and physicians, who make some use of the metric system as well.

AVOIRDUPOIS WEIGHT

16 ounces (oz.)	=	1 pound (lb.)
100 pounds	=	1 hundredweight (cwt.)
20 hundredweights	=	1 ton (T.)
2000 pounds	=	1 ton
2240 pounds	=	1 long ton
7000 grains (gr.)	=	1 pound avoirdupois

The *long ton,* sometimes called a *gross ton,* is used in selling iron and coal at the mines; it is also used by the United States Government in determining duty on merchandise taxed by the ton. Unless otherwise specified, a ton is considered to be 2000 pounds.

TROY WEIGHT

24 grains	=	1 pennyweight (pwt.)
20 pennyweights	=	1 ounce
12 ounces	=	1 pound
5760 grains	=	1 pound troy

In measuring the weight of precious minerals, such as gold, silver, and diamonds, troy weight is used instead of avoirdupois. Troy weight is also used by the United States Government in

weighing coins. In expressing the weight of precious stones, the *carat* is often used; a carat is equivalent to 3.168 grains troy, or 205.5 milligrams. The term *karat* is used to denote the fineness of gold, and means $\frac{1}{24}$ by weight of gold. For example, 24 karats fine means pure gold; 14 karats fine means $\frac{14}{24}$ pure gold by weight.

APOTHECARIES' WEIGHT

20 grains = 1 scruple (℈)	8 drams = 1 ounce (℥)
3 scruples = 1 dram (ʒ)	12 oz. = 1 pound (℔.)
5760 grains = 1 pound	

Although drugs and chemicals are bought and sold wholesale according to avoirdupois weight, druggists and physicians use apothecaries' weight in compounding medicines.

COMPARISON OF WEIGHTS

	Pound	*Ounce*
Troy	5760 grains	480 grains
Apothecaries'	5760 grains	480 grains
Avoirdupois	7000 grains	437½ grains

It should be noted that the grain is the same in all three systems, and that the troy and apothecaries' pound and ounce are respectively alike.

Metric Units of Weight

Compared to the variety of English units of weight the metric system is doubtless much simpler. Here the standard unit of weight is the *kilogram,* defined as the weight of a certain mass of platinum-iridium (kept at the International Bureau of Weights and Measures, near Paris) and known as the International Prototype Kilogram. The kilogram and the

gram are the two most widely used units, except for very large weights, where the metric ton is used.

METRIC MEASURES OF WEIGHT

1 centigram (cg.)	=	.01 gram
1 decigram (dg.)	=	.1 gram
1 kilogram (Kg.)	=	1000 grams
1 quintal (Q.)	=	100 kilograms
1 tonneau (T.)	=	1000 kilograms

The gram is equivalent to 15.432 grains, or about .002 pounds. Our five-cent piece weighs about 5 grams. The most common unit of weight in use is the kilogram, usually called *kilo* for short, which is equivalent to 2.2 pounds. The tonneau, usually called the metric ton, is thus equivalent to 2200 pounds.

APPROXIMATE EQUIVALENTS

1 ounce	=	28.35 grams
1 pound	=	454 grams = .45 kilograms
1 gram	=	.035 ounces
1 kilogram	=	2.2 pounds

EXAMPLES

1. A man's weight is recorded on his medical record as 82 Kg. How many lbs. does he weigh?

$$82 \times 2.2 = 180.4 \text{ lb.}$$

2. A 12-oz. loaf of bread is cut into 15 equal slices. What is the weight in grams of each slice?

$$\frac{12}{15} \times 28.35 = 22.7 \text{ gm.}$$

3. A photographic chemical costs \$4 per pound. How much is this per kilogram?

$$\$4 \div .45 = \$8.89$$

REFERENCE TABLE OF ACCURATE METRIC EQUIVALENTS

Length

1 inch = 2.540 centimeters		1 millimeter = .03937 inch	
1 foot = .3048 meter		1 centimeter = .3937 inch	
1 yard = .9144 meter		1 meter = 39.37 inches	
		1 meter = 3.281 feet	
1 rod = 5.029 meters		1 meter = 1.094 yards	
1 mile = 1.609 kilometers		1 kilometer = .6214 mile	

Area

1 sq. inch = 6.452 sq. cm.	1 sq. millimeter = .00155 sq. in.
1 sq. foot = .0929 sq. meter	1 sq. centimeter = .1550 sq. in.
1 sq. yard = .84 sq. meter	1 sq. meter = 1.196 sq. yd.
1 sq. rod = 25.29 sq. meters	1 sq. kilometer = .3861 sq. mi.
1 sq. mile = 2.589 sq. kilom.	1 hectare = 2.471 acres
1 acre = .4047 hectare	

Volume and Capacity

1 cu. in. = 16.3872 cu. centimeters	1 cu. centimeter = .06102 cu. in.
1 cu. ft. = .02832 cu. meter	1 cubic meter = 1.308 cu. yds.
1 cu. yd. = .7646 cu. meter	1 milliliter = .03381 liq. oz.
1 liquid qt. = .9464 liter	1 liter = 1.057 liq. qts.
1 liquid gal. = 3.785 liters	1 liter = .9081 dry qt.
1 dry qt. = 1.101 liters	1 hektoliter = 2.838 bushels
1 peck = 8.810 liters	1 dekaliter = 1.135 pecks
1 bushel = .3524 hektoliter	1 dekaliter = 2.642 liq. gals.

Weight

1 grain = .0648 gram	1 gram = 15.43 grains (troy)
1 oz. (avoir.) = 28.35 grams	1 gram = .03215 oz. (troy)
1 oz. (troy) = 31.10 grains	1 gram = .03527 oz. (avoir.)
1 lb. (avoir.) = .4536 kilog.	1 kilog. = 2.205 lbs. (avoir.)
1 lb. (troy) = .3732 kilog.	1 kilog. = 2.679 lbs. (troy)
1 ton (short) = .9072 met. ton	1 metric ton = 1.102 tons (short)

The three previous chapters constituted a thorough review of the basic elements of arithmetic. The next three chapters will present the most important aspects of algebra, geometry, and graphic methods. They will be sufficient for an understanding of the remainder of the book. However, if you remember your high school mathematics, you can probably pass on immediately to Part III. But if you are a bit dubious, it may be worth your while to read Chapters IV–VI first.

Some of the most useful parts of elementary algebra—the ideas and operations directly helpful in simple everyday problems—include the use of negative numbers, of algebraic symbols and expressions, and of formulas and equations.

Similarly, the most frequently used parts of elementary geometry, at least for practical purposes, include a knowledge of the common geometric figures and their properties; certain basic relations, such as formulas for areas and volumes; and certain basic concepts like similarity and symmetry.

Nor can the importance of graphic methods be overlooked. One can scarcely pick up a newspaper or a magazine these days without running across bar charts, line graphs, circle charts, pictograms and frequency distributions—all an essential part of our everyday means of exchanging ideas.

\$+50	\$+50	\$+50	\$+50	\$+50	\$+50	\$+50
−20	−30	−40	−50	−60	−70	−80
\$+30	\$+20	\$+10	\$ 0	\$−10	\$−20	\$−30

The "sum" (indicated below the line in each case) represents a *balance* if it is "+," and a *deficit* if it is "−"; in the one case where it is zero it is *neither* plus nor minus,—there simply isn't any balance.

From these two illustrations we can now formulate the rules for *adding* positive and negative numbers:

RULE 1: If the two numbers to be added both have the *same* sign, we find their *actual sum*, and prefix the same sign before the result; e.g.:

$+7$	-6	$+123$	-150
$+3$	-2	$+\ 56$	-125
$+10$	-8	$+179$	-275

RULE 2: If the two numbers to be added have *opposite* signs, we find their *actual difference*, and prefix the sign of the larger number before the result; e.g.:

-8	$-\ 4$	$+15$	$+12$
$+3$	$+10$	$-\ 4$	-20
-5	$+\ 6$	$+11$	$-\ 8$

Subtraction with Positive and Negative Numbers

Since subtraction is the reverse operation of addition, then subtracting +5 dollars is equivalent to adding −5 dollars; in other words, subtracting 5 positive dollars has the same effect as adding a debt of \$5. Or again; subtracting −8 dollars is equivalent to adding +8 dollars, since subtracting a debt of \$8 has the same effect as adding 8 positive dollars. This suggests the procedure for *subtracting* two "signed" numbers.

RULE: To subtract one signed number from another, change the sign of the subtrahend (mentally) and *add* the new subtrahend to the minuend in accordance with the rules for addition.

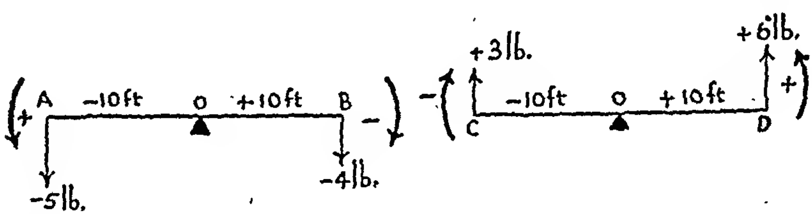
For example, subtracting in each case the second number from the first, we get:

+8	+6	+10	+ 8	−12	− 8	−15	−10
+3	+11	− 6	−12	+ 5	+18	− 5	−30
+5	− 5	+16	+20	−17	−26	−10	+20

Thus, to subtract +3 from +8, we think "−3 added to +8 gives +5"; to subtract +11 from +6, we think "−11 added to +6 gives −5"; etc.

Multiplication and Division with Signed Numbers

The rule for combining signed numbers, that is, positive and negative numbers, by multiplication is quite simple, and may be illustrated by the following diagrammatic device. Imagine a bar pivoted at O, and free to turn clockwise or counterclockwise, in response to forces applied at A, B, C or D. Let us agree that downward forces (such as at A and B) shall be called negative; then upward forces (such as at C and D) will be positive. Let us also agree that if the bar tends to turn clockwise, its rotation shall be called negative (−); and if it turns counterclockwise, its rotation is positive (+). Then, taking the four



possibilities suggested, we would have:

I. (−5 lb.) × (−10 ft.) = +50 lb.ft. turning tendency
II. (−4 lb.) × (+10 ft.) = −40 lb.ft. "
III. (+3 lb.) × (−10 ft.) = −30 lb.ft. "
IV. (+6 lb.) × (+10 ft.) = +60 lb.ft. "

Or, (−)×(−)=(+) and (−)×(+)=(−)
(+)×(+)=(+) and (+)×(−)=(−)

RULE: When two numbers with *like* signs are multiplied, their product is positive; if their signs are *unlike,* the product is negative.

The rule for dividing is exactly the same as for multiplication, which is just what we might have expected, since multiplication and division are inverse operations. Thus:

$$\text{Since } (+5)\,(+3) = +15, \text{ then } \frac{+15}{+3} = +5$$

$$\text{and } \frac{+15}{+5} = +3$$

$$\text{Since } (-5)\,(-3) = +15, \text{ then } \frac{+15}{-5} = -3$$

$$\text{and } \frac{+15}{-3} = -5$$

$$\text{Since } (+5)\,(-3) = -15, \text{ then } \frac{-15}{+5} = -3$$

$$\text{and } \frac{-15}{-3} = +5$$

$$\text{Since } (-5)\,(+3) = -15, \text{ then } \frac{-15}{-5} = +3$$

$$\text{and } \frac{-15}{+3} = -5$$

EQUATIONS AND FORMULAS

Equations in Algebra

An equation in algebra is merely a symbolic statement of the equality of two algebraic expressions. If this equality obviously holds true for *all* values of a letter, or of the variables, then the equation is called an *identity;* e.g.,

$$2a+3a=5a, \quad \text{or} \quad x+2y=2y+x$$

If the equality holds true for only *one* value of the variable (or for a limited number of values only), it is known as a *conditional equation;* e.g.,

$3x=12;$ $x/4=20;$ $2x+5=13;$ $x^2-x-12=0$

If the highest power of the variable (or the "unknown quantity") is one, we have a simple or first-degree equation, also called a *linear equation*. Such an equation has only one solution, or root; i.e., only one value will "satisfy" the equation, or make the equality true. If the highest power of the variable is two, then the equation is a *quadratic;* e.g., $3x^2=24$, or $x^2-5x=18$.

A quadratic equation always has two roots.

Solving Simple Equations by Multiplication or Division

Simple equations, i.e., first-degree equations, can often be solved by simply multiplying, or dividing, both "sides" of the equation by an appropriate number.

EXAMPLES

1. (a) Solve: $3x=27$
(Divide each side by 3)
$x=9$

(b) Solve: $100x=425$
(Divide each side by 100)
$x=4.25$, or $4\frac{1}{4}$

2. (a) Solve: $\frac{1}{5}x=13$
(Multiply each side by 5)
$x=65$

(b) Solve: $\frac{3x}{8}=7\frac{1}{2}$
(Multiply each side by $\frac{8}{3}$)
$x=20$

3. (a) Solve: $.04y=8$
(Multiply each side by $\frac{100}{4}$)
$y=200$

(b) Solve: $2\frac{1}{2}y=15$
(Multiply each side by $\frac{2}{5}$)
$y=6$

Solving Simple Equations by Addition or Subtraction

Sometimes a simple equation cannot be solved by multiplication or division; in that case we add or subtract an appropriate number to each side.

EXAMPLES

1. (a) Solve: $x+3=8$

(Subtract 3 from each side)

$$\begin{array}{r} x+3=8 \\ 3=3 \\ \hline x\ =5 \end{array}$$

(b) Solve: $y-5=2$

(Add 5 to each side)

$$\begin{array}{r} y-5=2 \\ 5=5 \\ \hline y\ =7 \end{array}$$

2. (a) Solve: $12=9+m$

(Subtract 9 from each side)

$$12-9=9+m-9$$
$$3=m$$
or $m=3$

(b) Solve: $40=x+15$

(Subtract 15 from each side)

$$40-15=x+15-15$$
$$25=x$$
or $x=25$

3. (a) Solve: $15=x-4$

(Add 4 to each side)

$$19=x$$
or $x=19$

(b) Solve: $5=12-y$

(Add y to each side)

$$5+y=12$$

(Subtract 5 from each side)

$$y=7$$

Simple Equations in General

As a rule, simple equations require a combination of two or more operations in order to find the root. There are short cuts possible, but we shall illustrate such solutions step by step.

EXAMPLES

1. Solve:	$3x+4=10$
(Subtracting 4)	$3x=6$
(Dividing by 3)	$x=2$, *Ans.*
2. Solve:	$5y+6=3y+18$
(Subtracting 6)	$5y=3y+12$
(Subtracting 3y)	$2y=12$
(Dividing by 2)	$y=6$, *Ans.*
3. Solve:	$6x-39=x-4$
(Adding 39)	$6x=x+35$
(Subtracting x)	$5x=35$
(Dividing by 5)	$x=7$, *Ans.*

What Is a Formula?

A mathematical formula is an equality which holds true for *all values* of the variables, or at least for all the values within certain limits, or for all values of a certain kind,—integers only. A formula involves two or more variables, which are usually designated by suggestive letters or conventional symbols.

In Chapter III we have already encountered four simple formulas, viz.:

Perimeter of a square:	$P=4s$
Perimeter of a rectangle:	$P=2l+2w$
Area of a square:	$A=s^2$
Area of a rectangle:	$A=lw$

Some New Formulas and How They Are Used

Many familiar relationships can be expressed by a mathematical formula.

EXAMPLES

1. The distance (D) traversed by an object moving at a constant speed (R) for a time (T) is expressed by the formula

$$D=R\times T.$$

(a) Find D when R=180 mi. per hr. and T=3½ hr.

(b) Find D when R=88 ft. per sec. and T=20 sec.

(a) $D=180\times 3\frac{1}{2}=630$ miles.

(b) $D=88\times 20=1760$ feet.

2. The familiar Fahrenheit temperature readings of a thermometer are related to the scientist's Centigrade scale as follows:

$$F=\frac{9}{5}(C)+32$$

Find F when C=100°; when C=20°; when C=0°.

$$F=\frac{9}{5}(100)+32=212°, \textit{Ans.}$$

$$F=\frac{9}{5}(20)+32=68°, \textit{Ans.}$$

$$F=\frac{9}{5}(0)+32=32°, \textit{Ans.}$$

3. The distance (S) in feet covered by a freely falling body in t seconds is given by the formula: $S=16t^2$. Find S when t=3; when t=10.

$$S=16(3)^2=16\times9=144 \text{ ft., } \textit{Ans.}$$
$$S=16(10)^2=1600 \text{ ft., } \textit{Ans.}$$

Using a Formula in Two Ways

In the relation $A=l\times w$, if we know l and w we can readily find A. But how can we find l if we know A and w? By inspection we know that we divide the area (A) by the width (w) to find the length (l); hence

$$l=\frac{A}{w}, \text{ and } w=\frac{A}{l}$$

Using a formula in more than one way is called "solving for another variable," or "changing the subject of the formula," and makes the use of formulas even more convenient.

EXAMPLES

1. From the formula D=RT, find an expression for R; an expression for T.

Dividing each side by T gives: $R=\frac{D}{T}$, *Ans.*

" " " " R gives: $T=\frac{D}{R}$, *Ans.*

2. Change the subject of the formula C=2.5 I, which expresses the relation between inches (I) and centimeters (C).

$$C=2.5\ I=\frac{5}{2}\ I$$

Multiplying each side by $\frac{2}{5}$ gives

$$I=\frac{2}{5}C, \text{ or } I=.4\,C, \textit{ Ans.}$$

3. The formula $I=\frac{E}{R}$ expresses the relation between current in amperes (I), volts (E), and resistance in ohms (R). Solve this formula for E; also for R.

Multiplying each side by R gives

$$E=IR, \textit{ Ans.}$$

Dividing both sides of the last equation by I gives

$$R=\frac{E}{I}, \textit{ Ans.}$$

4. Solve for C: $F=\frac{9}{5}C+32$.

$$\frac{9}{5}C=F-32, \text{ or } 9C=5(F-32)$$

$$C=\frac{5(F-32)}{9},$$

$$\text{or}\quad C=\frac{5}{9}(F-32), \textit{ Ans.}$$

5. Solve for B, and also for h: $A=\frac{1}{2}Bh$.

$$A=\tfrac{1}{2}Bh$$

$$2A=Bh$$

$$B=\frac{2A}{h}, \textit{ Ans.}$$

$$h=\frac{2A}{B}, \textit{ Ans.}$$

6. In the formula $S=\frac{1}{2}gt^2$, solve for g; for t.

$$S=\tfrac{1}{2}gt^2$$

$$2S=gt^2$$

$$g=\frac{2S}{t^2}, \textit{ Ans.}$$

$$\text{also,}\quad t^2=\frac{2S}{g}, \text{ and } t=\sqrt{\frac{2S}{g}}, \textit{ Ans.}$$

PROBLEMS FOR PRACTICE: NO. 24

1. Solve for x: $x/9=1\frac{2}{3}$.
2. If $I=.045\ Pt$, find I when $P=\$500$ and $t=\frac{2}{3}$.
3. If $I=P\times R\times T$, solve for P; for R.
4. If $L=2\pi Rh$, solve for R.
5. If $E=\frac{1}{2}\ mv^2$, find E when $m=150$ lb. and $v=20$ ft. per sec.
6. Solve for y: $2y-18=15-3\frac{1}{2}y$.
7. If $w_1d_1=w_2d_2$, find d_2 when $w_1=60$, $d_1=8$, and $w_2=96$.
8. Solve for r: $I=\dfrac{E}{R+r}$.
9. If $A=\frac{1}{2}h(B+b)$, find B.
10. In the formula $A=\frac{1}{2}\times b\times h$, how does A change:
    (a) when b remains the same, but h is tripled?
    (b) when b and h are both halved?
    (c) when b is doubled and h is tripled?

CHAPTER V

GEOMETRY

PROBABLY the first geometric shapes which primitive man recognized as such were the cylinder and the circle. These he observed in the common objects about him—the trunk of a tree, his crude implements and his everyday utensils. The generalization of other figures—triangles and rectangles—did not presumably take place until many thousands of years later

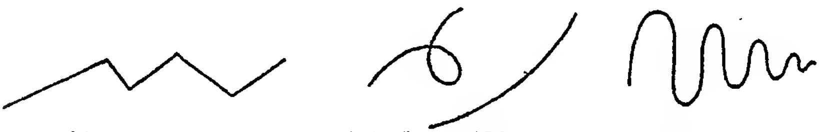
Man's use of geometric knowledge for purposes of measurement was reasonably developed among the Babylonians. The Egyptians made considerable use of it in surveying the land near the River Nile, which overflowed its banks periodically, and necessitated the re-establishment of boundary lines.

Practical geometry deals with the nature and properties of various geometric forms, such as rectangles, triangles, circles, etc., and emphasizes especially the measurement of such figures. Hence the chief value of practical geometry is in problems of design and construction. No towering skyscraper could be built, no bridge could span a river, no airplane could take to the skies, no ship could sail the seas, were it not for practical geometry. Indeed, precision measurements, machine tools and parts, and modern industrial production could never have become as highly developed as at present were it not for the application of geometrical and trigonometrical principles.

COMMON GEOMETRIC FORMS

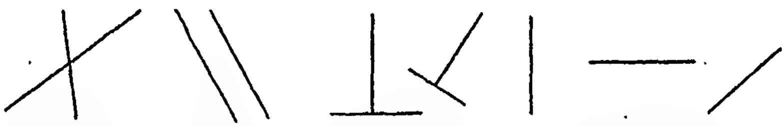
Lines and Angles

Geometric lines are fictions in the sense that, while we draw them, we think of them as having no *width,* simply length. Lines may be curved or straight. A straight line is illustrated by an edge of a cube or a crease in a folded paper. It is the shortest distance between any two points. Straight lines either



BROKEN LINE CURVED LINES WAVY LINE

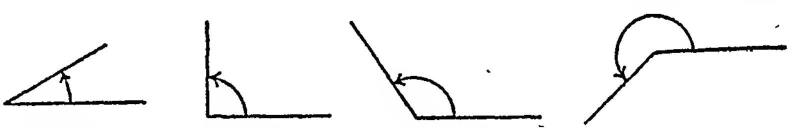
intersect or they don't; if two straight lines intersect, they meet at one point only; if they are parallel, they never meet however far prolonged in either direction. Two parallel lines are every-



INTERSECTING LINES PARALLEL LINES PERPENDICULAR LINES VERTICAL HORIZONTAL OBLIQUE

where equally distant from one another.

An angle is conveniently regarded as the figure formed by the rotation of a line around a point as a pivot. One complete rotation represents 360°; one quarter of a turn is a 90°-angle, or a right angle. Angles of less than 90° are acute angles; those of more than 90°, but less than 180°, are obtuse angles.



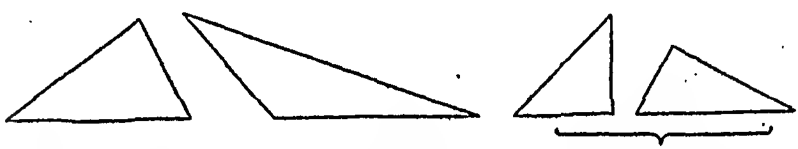
ACUTE ANGLE RIGHT ANGLE OBTUSE ANGLE REFLEX ANGLE

Angles greater than 180° are called reflex angles. In mechanics we commonly think of angles of more than 360°; for example, in

connection with rotating wheels and the like. The sides of an angle may be extended indefinitely; the lengths of the sides have nothing to do with the size of the angle, which is the amount of rotation, expressed by the number of degrees of turning.

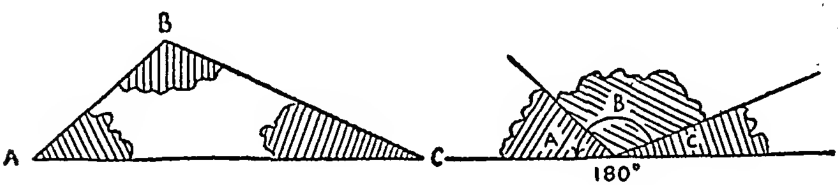
Triangles

The simplest figure consisting of straight lines only and which can completely enclose a plane (flat) surface is a three-sided figure, or triangle. Every triangle has three sides, three angles, and three vertices; the vertices are the points at which the sides meet. Triangles may be described in terms of their angles,

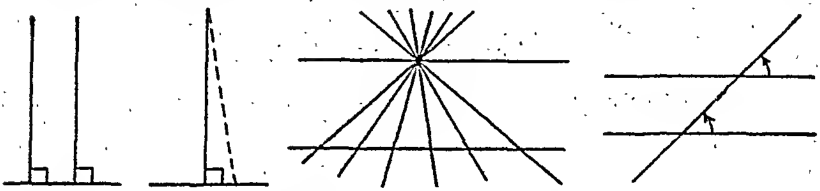


ACUTE TRIANGLE OBTUSE TRIANGLE RIGHT TRIANGLES

as acute triangles, obtuse triangles, or right triangles. The sum of all three angles of a triangle is exactly 180°, regardless of the size or shape of the triangle. This can easily be verified, in an

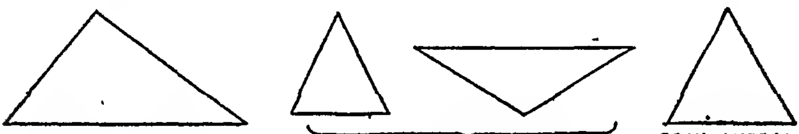


approximate way, by tearing off the corners of a triangular piece of cardboard and fitting them together, as shown, to form an angle of 180°. The fact that the sum of the angles equals 180° is very important and very useful. It explains, for example, why no triangle can have more than one right angle or obtuse angle; why two lines both perpendicular to a third line must be parallel to each other; why only one line can be drawn perpendicular to another through a given point; why only one line can be drawn parallel to a given line through a point

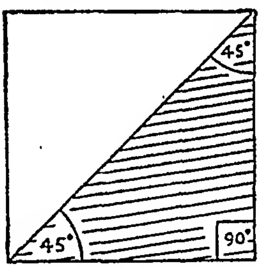


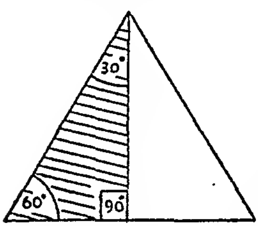
outside; why a transversal to two parallel lines makes equal angles with them; and many other important relationships.

Triangles may also be classified according to the relative lengths of their sides. If all three sides are of equal length, it is an equilateral triangle; if two sides are equal, it is an



SCALENE TRIANGLE ISOSCELES TRIANGLES EQUILATERAL TRIANGLE





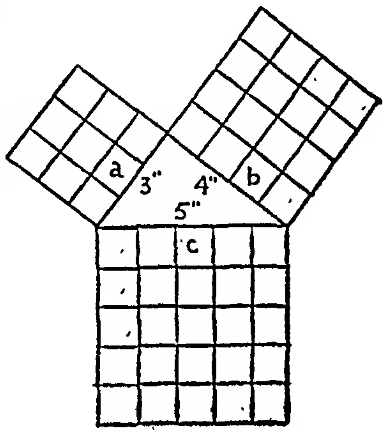
isosceles triangle; if all three sides are of different lengths, it is a scalene triangle. The diagonal of a square divides the square into two isosceles right triangles. The two angles opposite the equal sides of an isosceles triangle are always equal to each other; in the case just mentioned, these angles are each 45°. Another special triangle commonly encountered is a 30°–60°–90° triangle, which is half of an equilateral triangle; in such a triangle the longest side is always twice as long as the shortest side. In an equilateral triangle, all three angles are equal, each of them being 60°.

Draftsmen's *triangles* are drawing instruments made of flat pieces of celluloid or xylonite. They come in two common shapes—the 45°–45°–90° triangle, and the 30°–60°–90° triangle, and are among the draftsman's most practical and frequently used tools.

Right Triangle Rule

One of the most famous relationships in all geometry concerns right triangles. It is named after Pythagoras, a famous Greek mathematician who lived a little more than two thousand years ago. This is the relationship: the square of the side opposite the right angle (hypotenuse) is equal to the sum of the squares of the other two sides; or, $c^2=a^2+b^2$. It is illustrated here for small whole numbers, but it holds true for any right triangle, whatever the lengths of its sides may be. In other words, if we know any two sides of a right triangle, we may use this relation to find the third side.

$$c=\sqrt{a^2+b^2};\ a=\sqrt{c^2-b^2};\ b=\sqrt{c^2-a^2}$$

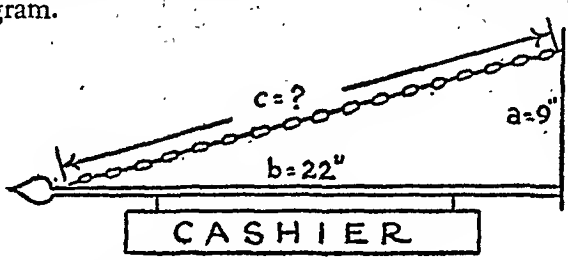


This so-called Pythagorean relationship has an extremely interesting history. It was known to the ancient Egyptian "rope-stretchers," who used it, when surveying, to determine perpendicular lines. For this purpose they used a rope which was knotted at equal intervals. By stretching the rope around stakes driven into the ground in such a way as to form a triangle having sides of 3, 4 and 5 intervals in length, respectively, they were able to determine a perfect right angle, or two mutually perpendicular directions.

The Egyptians as well as other ancient peoples were quite interested in being able to lay out perpendiculars in this manner. For one thing, they found considerable occasion for land measuring. The farm lands in the fertile Nile valley required frequent resurveying to settle boundary disputes, which arose from the fact that the River Nile overflowed its banks periodically, obliterating all boundaries for miles on either side.

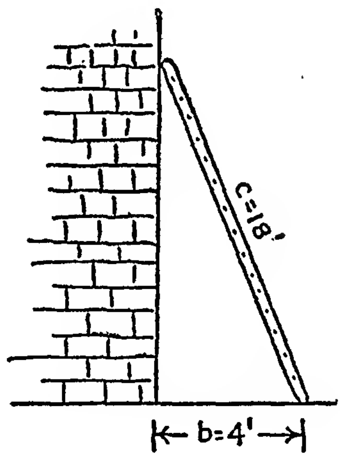
EXAMPLES

1. Find the length of chain required to support the sign shown in the diagram.



$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{(9)^2 + (22)^2}$$
$$c = \sqrt{81 + 484}$$
$$c = \sqrt{565} = 23.8, \textit{ Ans.}$$

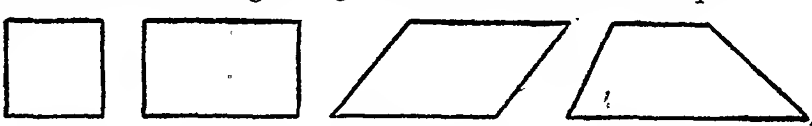
2. An 18-foot ladder rests against a wall. If the foot of the ladder is 4 feet from the base of the wall, how high a point on the wall will the ladder reach?



$$a = \sqrt{c^2 - b^2}$$
$$a = \sqrt{(18)^2 - (4)^2}$$
$$a = \sqrt{324 - 16}$$
$$a = \sqrt{308} = 17.6, \textit{ Ans.}$$

Quadrilaterals

All four-sided straight-line figures are called quadrilaterals. The commonest of these are squares and rectangles, both of which contain all right angles. The four sides of a square and

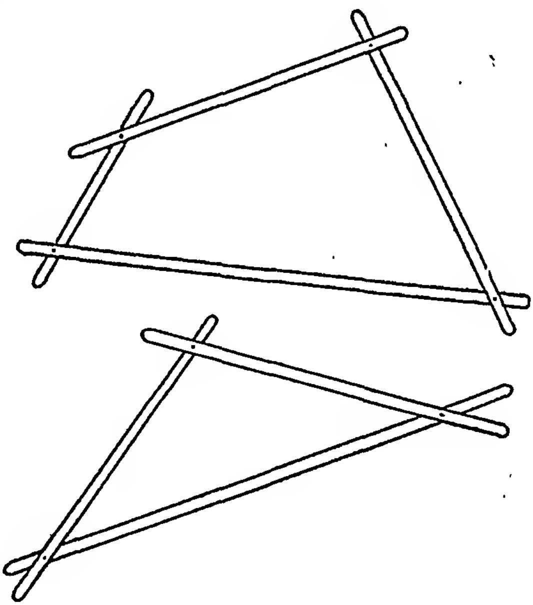


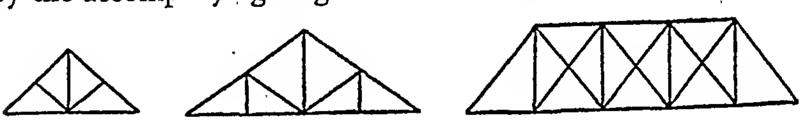
SQUARE RECTANGLE PARALLELOGRAM TRAPEZOID

the opposite sides of a rectangle are equal in length. A parallelogram is a quadrilateral whose opposite sides are parallel and

equal, but whose angles are oblique (i.e., not right angles). The opposite angles in a parallelogram are equal; one pair is acute, the other, obtuse. The diagonals of a square and a rectangle are respectively equal in length; in a parallelogram, however, one diagonal is always longer than the other. If only one pair of opposite sides in a quadrilateral is parallel, the figure is called a trapezoid, and the two parallel sides are called its bases.

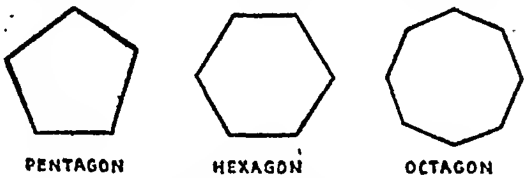
Quadrilaterals are not "rigid" figures; i.e., they can be readily "deformed." This means that the shape of a quadrilateral can be changed, by changing its angles, without altering the lengths of its sides. This will be understood by considering four sticks fastened together with nails, as shown; a child knows that such a contrivance can be "pushed out of shape" quite easily. But this is not so with a triangle, which is a rigid figure. Once the lengths of its three sides have been determined, its angles also are automatically determined and cannot be altered. It has a fixed shape. This rigidity gives a triangular frame or structure far more mechanical strength than a four-sided frame, a fact of great practical value in mechanical construction and design, as suggested by the accompanying diagrams of trusses.





Regular Polygons

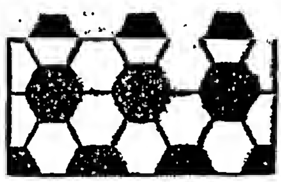
Any figure of straight lines having three or more sides is called a polygon. Some pleasing shapes used in ornament and design are regular polygons like the pentagon (5 sides), hexagon (6 sides), and octagon (8 sides). A polygon is regular if all

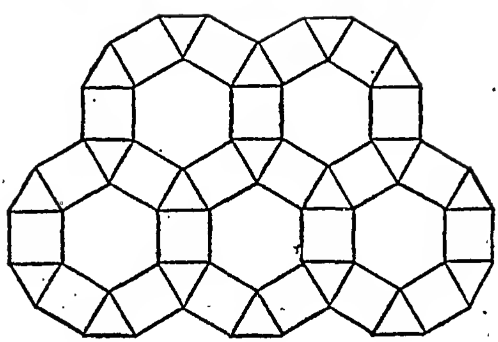


REGULAR POLYGONS

of its sides are equally long, *and* all of its angles are equal to one another. A polygon can have equal sides without having equal angles, and vice versa.

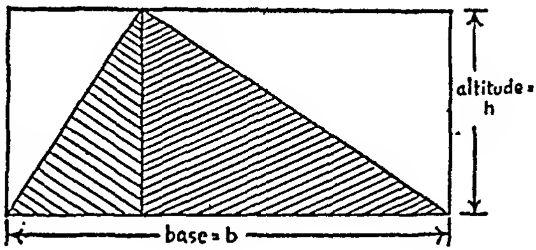
The following sketches suggest how regular polygons are utilized in ornamental design, such as in tile-work, linoleum patterns, and so on.



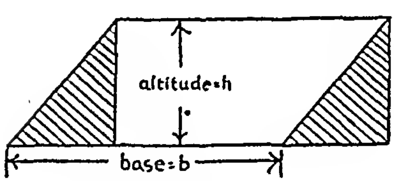


Areas

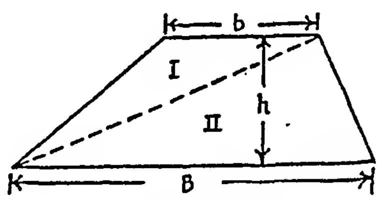
The area of a square, as we have already seen, is equal to the square of the length of its side; the area of a rectangle equals the product of its length and its width. The area of a triangle may readily be seen to be equal to half that of a rectangle having the same base and altitude, respectively, as the triangle; or, Area of Triangle= $\frac{1}{2}b \times h$.



The area of a parallelogram, like that of a rectangle, simply equals the product of its base by its altitude; however, the *altitude* of a parallelogram must not be confused with the length of the shorter side, since they are not the same, as in the case of a rectangle.



AREA = b × h

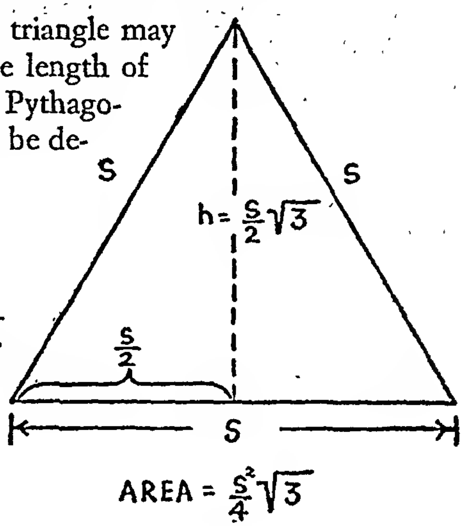


AREA = $\frac{1}{2}h\,(B+b)$

If a four-sided figure has only *one* pair of sides parallel, it is called a *trapezoid*. The area of a trapezoid may be found by adding together the areas of the two triangles into which it is divided by either diagonal. These two triangles each have the same altitude (viz., the altitude of the trapezoid), but different bases; thus

$$\Delta\mathrm{I}=\tfrac{1}{2}b\times h$$
$$\Delta\mathrm{II}=\tfrac{1}{2}B\times h$$
$$\text{Trapezoid}=\tfrac{1}{2}b\times h+\tfrac{1}{2}B\times h=\tfrac{1}{2}(B+b)\times h.$$

The area of an equilateral triangle may be found if we are given the length of one of its sides. For by the Pythagorean relation, its altitude can be determined:



$$h^2=s^2-\frac{s^2}{4},$$

or $h^2=\dfrac{3s^2}{4}$, $h=\sqrt{\dfrac{3s^2}{4}}=\dfrac{s}{2}\sqrt{3}.$

Then, area $=\frac{1}{2}\times s\times\dfrac{s}{2}\sqrt{3}$,

or area $=\dfrac{s^2}{4}\sqrt{3}.$

EXAMPLES

1. How many sq. ft. of board are needed for 8 shelves, each 9″ wide and 6′3″ long?

   $\frac{3}{4}\times\frac{25}{4}\times 8=37\frac{1}{2}$ sq. ft., *Ans.*

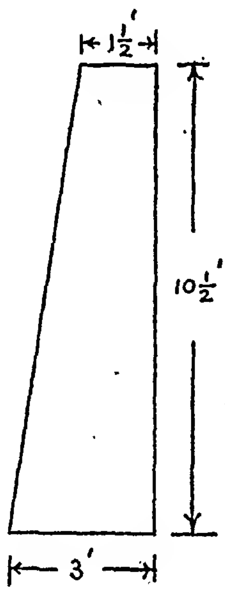
2. How many sq. in. in the surface of a triangular sheet of metal, if the base is $20\frac{1}{2}$″ and the altitude is $8\frac{1}{2}$″?

   $\frac{1}{2}\times\frac{41}{2}\times\frac{17}{2}=87.1$ sq. in., *Ans.*

3. What is the cross-sectional area of a retaining wall with dimensions as shown?

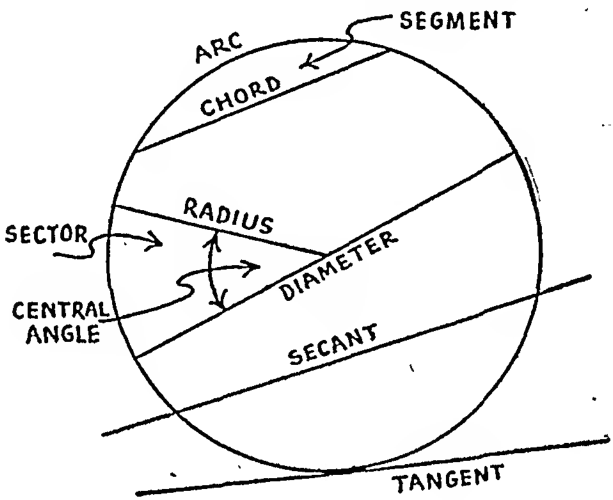
   $A=\frac{1}{2}\times\frac{21}{2}\times(1\frac{1}{2}+3)$

   $\frac{1}{2}\times\frac{21}{2}\times\frac{9}{2}=23.6$ sq. ft., *Ans.*



Circles

Geometrically, a circle is a closed curved line, every point of which is the same distance from a point within called the *center*. The length of such a curve is called the *circumference;* any portion of the circle is called an *arc*. A straight line which



touches a circle at one point only is called a *tangent;* if a straight line cuts a circle at two points, it is called a *secant;* if a line segment terminates in the circle, it is a *chord*. The portion of a circle "cut off" by a chord is called its *subtended arc,* and the area enclosed by a chord and its subtended arc is known as a *segment* of the circle. The longest chord of a circle passes through its center and is called the *diameter;* half the diameter is a *radius*. All radii of a circle branch out, like the spokes of a wheel, from the center; naturally they are all equal in length. The angle formed by two radii is known as a *central angle,* and the area included by two radii and their subtended arc is called a *sector* of the circle (a wedge of apple pie).

The ratio of any circumference to its diameter is constant,

whether the circle is the size of a penny or a cartwheel. This ratio is called π (pi). Numerically, its value is approximately $3\frac{1}{7}$; or, a circumference is about $3\frac{1}{7}$ times as long as its diameter. In symbols,

$$\frac{C}{D}=\pi=3\tfrac{1}{7}=3.1416\ldots\ldots$$

This relation enables us to find the circumference if the radius or diameter is known, for $C=\pi D=2\pi R$.

The "area of a circle," or more correctly, the area enclosed by a circle is given by the expressions: $A=\pi R^2$, or $A=\frac{1}{4}\pi D^2$.

The area of a sector is the same proportional part of the entire area of the circle as its central angle is of 360°; or,

$$\frac{\text{Area of sector}}{\text{Area of circle}}=\frac{\text{Angle of sector}}{360^\circ}$$

EXAMPLES

1. Find the circumference of a circle whose radius is 4 ft. 8 in.

$$4 \text{ ft. } 8 \text{ in.}=4\tfrac{2}{3} \text{ ft.}$$

$$C=2\pi R=2\times\tfrac{22}{7}\times\tfrac{14}{3}=\tfrac{88}{3}=29\tfrac{1}{3} \text{ ft., } \textit{Ans.}$$

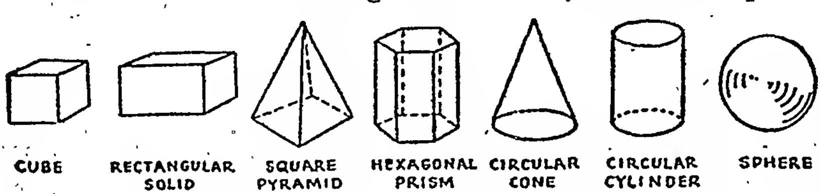
2. What is the area of a circle whose diameter is $10\frac{1}{2}$ inches?

$$A=\pi R^2$$

$$A=\tfrac{22}{7}\times\tfrac{21}{4}\times\tfrac{21}{4}=86.6 \text{ sq. in., } \textit{Ans.}$$

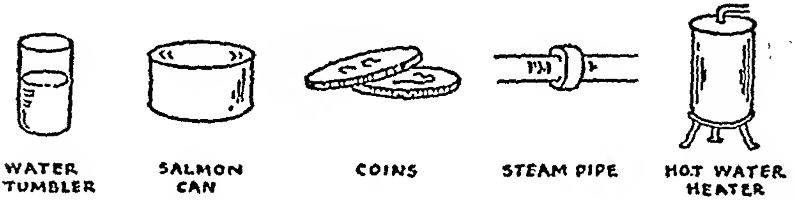
Solid Geometric Forms

The commonest geometric forms with plane surfaces are the "cube" and the "rectangular solid." Pyramids and prisms



(other than rectangular prisms) are not encountered frequently among common everyday objects. Solids with part of their

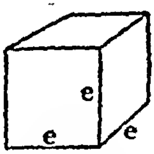
surface curved, like the cone and the cylinder, are met with more often, especially the cylinder.



Surface and Volume of Rectangular Solids

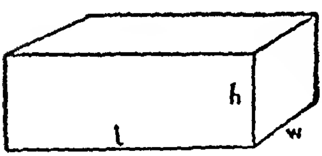
The total surface of a rectangular solid consists of the areas of three pairs of equal rectangles: the two side faces, the two end faces, and the top and bottom. Or,

$$A=2lw+2lh+2wh$$



In the case of the cube, the area becomes

$$A=6e^2$$



The volume of a rectangular solid is $V=lwh$; for a cube the volume is $V=e \times e \times e$, or $V=e.^3$

EXAMPLES

1. Find the total surface of the walls and ceiling of a room (disregarding windows, doors, etc.) if its dimensions are $21' \times 14'$, and the ceiling height is $10'$.

Long walls:	$2 \times 21 \times 10 = 420$	sq. ft.
End walls :	$2 \times 14 \times 10 = 280$	" "
Ceiling :	$21 \times 14 \quad = 294$	" "
	$\overline{994}$	sq. ft., *Ans.*

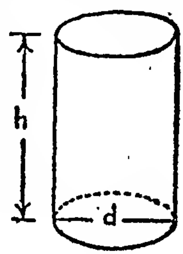
2. What is the capacity in cubic inches of an open cubical box $1\frac{1}{2}$ feet on an edge?

$$1\tfrac{1}{2}'=18''$$

$V=e^3=18 \times 18 \times 18=5832$ cu. in., *Ans.*

Cylinders

A form like a tomato can is known as a *right circular cylinder*. The *altitude* of such a cylinder is the perpendicular distance between its two flat surfaces, known as *bases*. A *disc* is a right circular cylinder whose altitude is comparatively small compared to the diameter of its bases; e.g., a coin or a poker chip. If the altitude is many times longer than the diameter, the cylinder is also called a *rod* or a *tube*, depending upon whether it is "solid," or "hollow." The area of that portion of the surface of a cylinder that is curved is called its "lateral area" (L); the entire surface, including the two bases, is called the "total area" (T). The formulas for finding the surface and volume, respectively, of a right circular cylinder are as follows:



$$L=2\pi Rh=\pi Dh$$
$$T=2\pi Rh+2\pi R^2=2\pi R\,(h+R)$$
$$V=\pi R^2h$$

EXAMPLES

1. Find the total surface of a cylindrical metal can, 21″ high and 14″ in diameter.

$$T=2\times{}^{22}/_{7}\times7\times21+2\times{}^{22}/_{7}\times7\times7$$
$$=924+308=1232 \text{ sq. in., } \textit{Ans.}$$

2. What is the capacity in cubic centimeters of a cylindrical glass jar 10.5 cm. in diameter and 42 cm. tall?

$$V=\pi R^2h$$
$$={}^{22}/_{7}\,({}^{10.5}/_{2})\,({}^{10.5}/_{2})\,(42)=3638.25 \text{ cubic centimeters, } \textit{Ans.}$$

PROBLEMS FOR PRACTICE: NO. 25

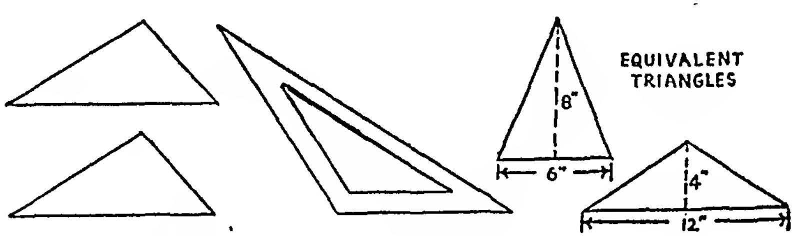
1. A cylindrical tin can measures 16″ in diameter and 21″ in height. Allowing ½″ for overlapping, how many square inches of paper are required for a label covering the curved surface completely?
2. What is the area of a sector of a circle whose central angle is 40°, if the diameter of the circle is 28″?

3. Find the area of a cross-section of a piece of timber in the form of a trapezoid, the bases of the trapezoid being $4\frac{1}{2}''$ and $6\frac{1}{2}''$ and the altitude $5''$.
4. The area of a triangular show card is 648 sq. in. If its base is $48''$, what is its altitude?
5. What is the length of the diagonal of a baseball diamond, which is a square 90 feet on a side?
6. Find the diameter of a circular flower bed whose area is 2464 sq. ft.
7. A cube of metallic gold $2''\times 2''\times 1''$ is rolled into sheets $5''$ square and $0.01''$ thick. How many such sheets will there be?
8. The largest possible disc is stamped from a square metal plate $14''$ on a side. What per cent of the metal is wasted?

SIMILAR FIGURES

Congruence and Similarity

If two geometric figures have the same shape and the same size they are *congruent* figures. If two figures have the same shape, but differ in size, they are *similar* figures. If they have the same size, but differ in shape, they are said to be *equivalent* figures.



CONGRUENT TRIANGLES SIMILAR TRIANGLES

Properties of Similar Figures

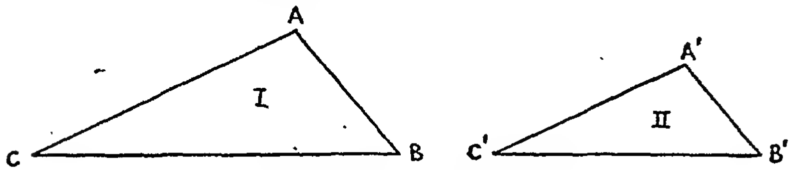
When two geometric figures are similar, the following two properties are always found to hold:

(1) their corresponding angles are respectively equal.
(2) their corresponding sides are respectively proportional.

These two conditions may be taken as the definition of similar figures. Thus in the accompanying figure, if triangle I is similar to triangle II, it follows that:

(1) $\measuredangle A = \measuredangle A'$; $\measuredangle B = \measuredangle B'$; $\measuredangle C = \measuredangle C'$;

(2) $\dfrac{AC}{A'C'} = \dfrac{AB}{A'B'} = \dfrac{BC}{B'C'}$.



Conditions for Similarity

It can be shown, furthermore, that any two triangles will be similar if any *one* of the following conditions prevail:

(1) if the three angles of one triangle are equal, respectively, to the three angles of the other.
(2) if any two angles of one triangle are equal, respectively, to the corresponding angles of the other.
(3) if the triangles are right triangles, and an acute angle of one equals an acute angle of the other.
(4) if the six sides of the two triangles are in proportion.
(5) if one angle of one triangle equals an angle of the other, and the four sides including these two angles are in proportion.

It is easily seen, therefore, that all squares are similar to one another, all equilateral triangles are similar, and, in fact, any two *regular* polygons having the same number of sides are similar to each other. Similar figures are frequently encountered. All plans drawn to scale, and all working drawings and working models are examples of similar geometric figures, plane or solid. So is a diorama; and, if we compare a photograph with the plane image on the retina, we have another

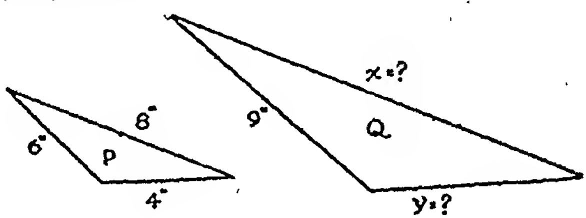
example of similar figures. Enlargements and reductions of photographs, pictures, etc., are also similar to the originals.

Use of Proportion with Similar Figures

One of the chief practical uses of similar figures is that of computing unknown parts from known parts by means of a proportion. Thus triangles P and Q are similar, and if we know that the side corresponding to 6″ in triangle P is 9″ in triangle Q, then we know that since *all* the sides of these two triangles are in proportion,

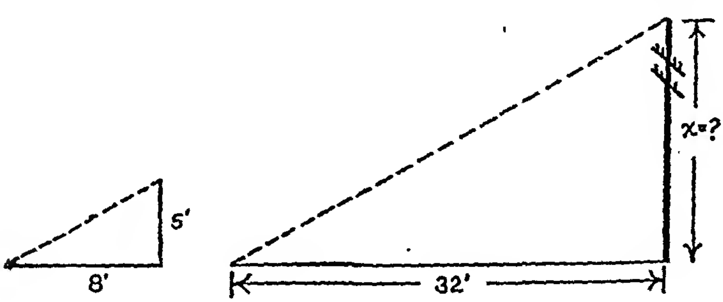
$$\frac{6}{9}=\frac{8}{x}, \text{ or } x=12;$$

and $\frac{y}{4}=\frac{9}{6}$, or $y=6$.



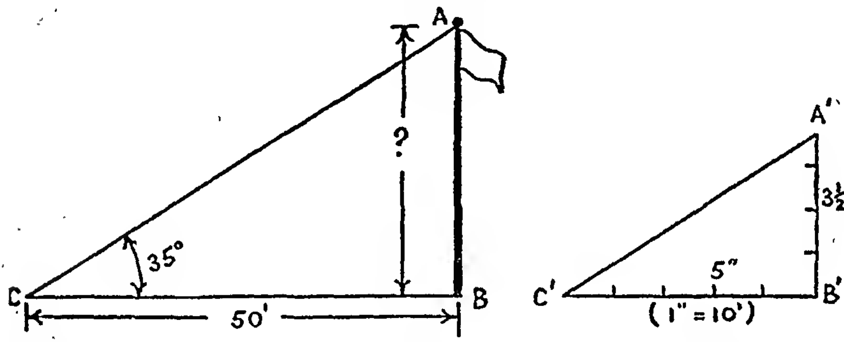
Indirect Measurement

This principle of applying a proportion to similar figures is rather useful in measuring inaccessible distances "indirectly," such as the height of a mountain, the width of a river, and so on. The method was used by the ancient Greeks, and has been the basis for surveying through the ages. In one form it was called "shadow-reckoning." For example, suppose that when the shadow of a pole is 32 ft., a boy 5 ft. tall casts a shadow 8 ft. long. By considering the two right triangles shown, and



recalling that the angle formed by the sun's rays with the earth is the same for the boy as it is for the pole, we see that the triangles are similar, and that therefore $\frac{x}{32}=5/8$, or $x=20$ ft.

A variation of the method, but using the same principle, is the following. We wish to find the height of the flagpole AB, and, by using a simple instrument to measure the angle at C, and measuring the distance along the ground from C to B, we have enough data. Suppose the angle at C upon measurement is found to be equal to 35°, and CB is 50 ft. We then

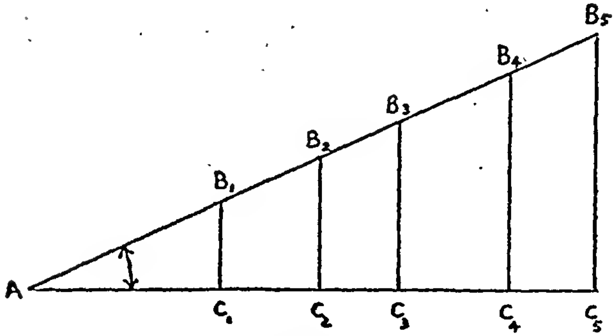


draw a right triangle A′B′C′ to scale, making it similar to triangle ABC; i.e., $\measuredangle C=\measuredangle C'=35°$, and, using a scale of $1''=10'$, we complete triangle A′B′C′. Then, measuring the length of A′B′, we find it to be 3½″; but since $1''=10'$ on our scale-drawing (triangle A′B′C′), then AB, in the other similar triangle, must, of course, be 35 ft.

The Tangent Ratio

If you study the accompanying diagram, you will realize that in this series of similar right triangles, AB_1C_1, AB_2C_2, AB_3C_3, AB_4C_4 and AB_5C_5, angle A is common to all. Moreover, since

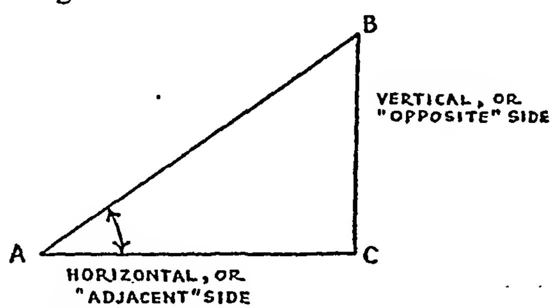
$$\frac{B_1C_1}{AC_1}=\frac{B_2C_2}{AC_2}=\frac{B_3C_3}{AC_3}=\frac{B_4C_4}{AC_4}=\frac{B_5C_5}{AC_5}$$



then angle A must be measured by a "constant ratio" in terms of the two sides. If, for example, $B_1C_1=2$, and $AC_1=4$, then the series of ratios might run like this:

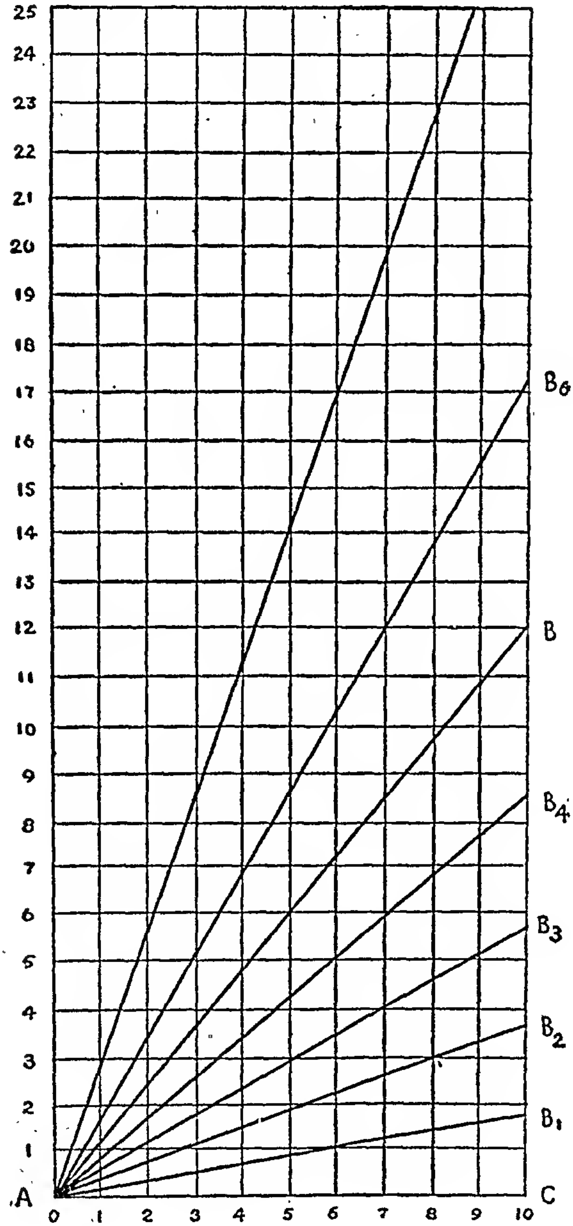
$$2/4=3/6=4/8=6/12=7/14;$$

but the *numerical* value of these is constant, and equal to 0.5. This ratio for an acute angle, viz., the $\frac{\text{"vertical side"}}{\text{"horizontal side"}}$, or the $\frac{\text{side opposite the angle}}{\text{side adjacent to the angle}}$, is called the tangent ratio, or simply the *tangent* of the angle. This is written: tangent of $A=\frac{\text{length of BC}}{\text{length of AC}}$, or simply, $\tan A=\frac{BC}{AC}$.



Further reflection will show that as an angle increases or decreases, its tangent also increases or decreases. Every angle has a specific tangent value. The numerical value of the tangent

of each angle can be obtained from a scale drawing like the one shown here.



The base line AC is 10 units. With a protractor, angles are drawn as follows:

∢ $CAB_1 = 10°$;
∢ $CAB_2 = 20°$;
∢ $CAB_3 = 30°$;
∢ $CAB_4 = 40°$;

etc. Then, measuring the respective lengths of B_1C, B_2C, B_3C, etc., we can draw up a table as follows, which the reader can readily verify.

Angle	*Degrees*	*Side opposite*	*Side adjacent*	$\frac{BC}{AC}$	*Tan A*
CAB_1	10°	1.8	10	.18	tan 10° = .18
CAB_2	20°	3.6	10	.36	tan 20° = .36
CAB_3	30°	5.8	10	.58	tan 30° = .58
CAB_4	40°	8.4	10	.84	tan 40° = .84
CAB_5	50°	11.9	10	1.19	tan 50° = 1.19
CAB_6	60°	17.3	10	1.73	tan 60° = 1.73

A complete table of tangents of angles from 1° to 89° is given for reference, and for use in connection with problems, as will now be illustrated.

TABLE OF TANGENTS

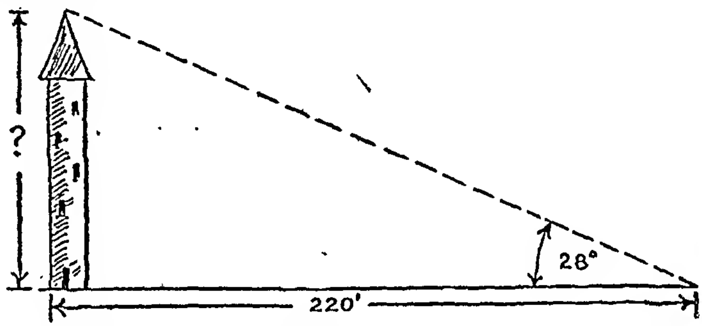
Angle	*Tangent*	*Angle*	*Tangent*	*Angle*	*Tangent*	*Angle*	*Tangent*	*Angle*	*Tangent*
1°	.02	19°	.34	37°	.75	55°	1.43	73°	3.27
2°	.03	20°	.36	38°	.78	56°	1.48	74°	3.49
3°	.05	21°	.38	39°	.81	57°	1.54	75°	3.73
4°	.07	22°	.40	40°	.84	58°	1.60	76°	4.01
5°	.09	23°	.42	41°	.87	59°	1.66	77°	4.33
6°	.10	24°	.44	42°	.90	60°	1.73	78°	4.70
7°	.12	25°	.47	43°	.93	61°	1.80	79°	5.14
8°	.14	26°	.49	44°	.96	62°	1.88	80°	5.67
9°	.16	27°	.51	45°	1.00	63°	1.96	81°	6.31
10°	.18	28°	.53	46°	1.03	64°	2.05	82°	7.12
11°	.19	29°	.55	47°	1.07	65°	2.14	83°	8.14
12°	.21	30°	.58	48°	1.11	66°	2.25	84°	9.51
13°	.23	31°	.60	49°	1.15	67°	2.36	85°	11.43
14°	.25	32°	.62	50°	1.19	68°	2.48	86°	14.30
15°	.27	33°	.65	51°	1.23	69°	2.61	87°	19.08
16°	.29	34°	.67	52°	1.28	70°	2.75	88°	28.64
17°	.31	35°	.70	53°	1.33	71°	2.90	89°	57.29
18°	.32	36°	.73	54°	1.38	72°	3.08		

Using a Tangent Table

Many simple practical problems in indirect measurement can be easily solved by means of a tangent table.

EXAMPLES

1. An observer notes that the "angle of elevation" of the top of a tower is 28°. From where he is standing to the base of the tower is 220 ft. How high is the tower?

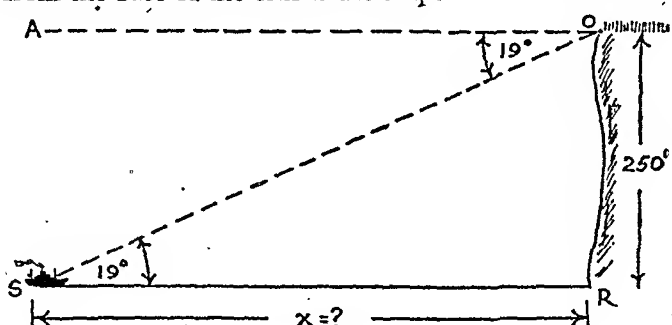


Calling the height of the tower x; then $\tan 28° = \dfrac{x}{220}$

From the table, $\tan 28° = .53$

Then $\dfrac{x}{220} = .53$, or $x = (.53)\,(220) = 116.6$, *Ans.*

2. From the top of a cliff 250 ft. high, an observer finds that the "angle of depression" ($\measuredangle$AOS) of a ship at sea is 19°. How far out from the base of the cliff is the ship?



$\measuredangle AOS = \measuredangle RSO$ (Why?)

$\tan 19° = .34$

hence $\dfrac{250}{x} = .34$

or $.34x = 250$

$x = 735.3$ ft., *Ans.*

Using the Right Triangle Rule in Connection with the Tangent of an Angle

If we know one side of a right triangle and one of its acute angles, we can find the other side and the hypotenuse.

EXAMPLE

Suppose that in right triangle RST, $\measuredangle R = 37°$ and $RS = 21''$. Find TS and RT.

From the tangent table, $\dfrac{TS}{RS} = \tan 37°$, or $\dfrac{TS}{21} = .75$, and $TS = (21)(.75) = 15.8$, or approx. $16''$.

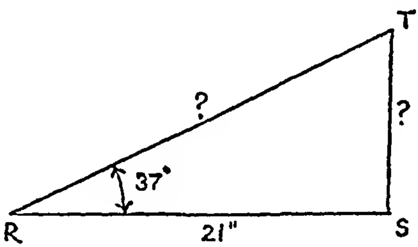
By the right triangle rule,

$(RT)^2 = (RS)^2 + (ST)^2$

or, $(RT)^2 = (21)^2 + (16)^2$

$(RT)^2 = 441 + 256 = 697$

$RT = \sqrt{697} = 26.4$, *Ans.*



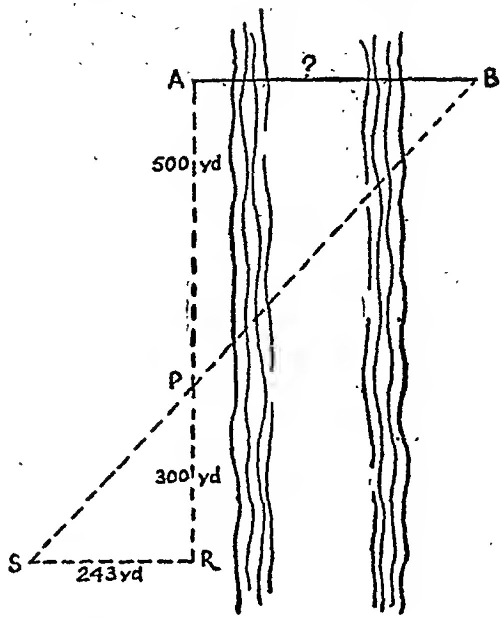
Other Methods of Using Similar Triangles

Inaccessible distances may also be measured by the use of similar triangles with the use of a table of tangents, as suggested by the two following illustrations.

EXAMPLES

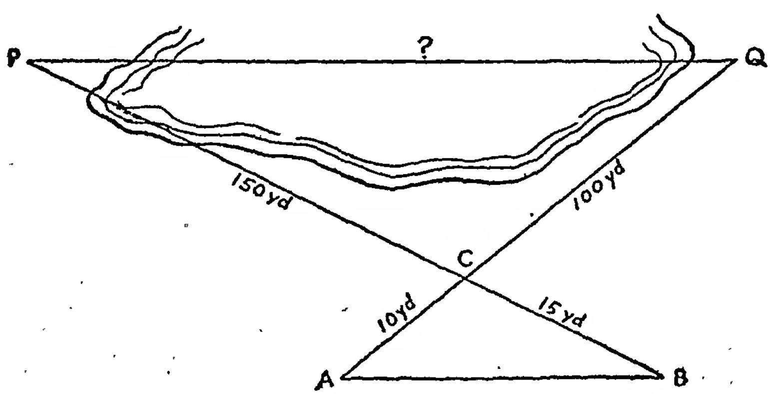
1. To find the distance AB across a river.

Lay off a "base line" AR, and a line RS perpendicular to AR. From S, sight a line to B, which intersects AR at P. Measuring, AP=500 yd., PR=300 yd., and RS=243 yd. Then, since triangle ABP is similar to triangle PRS, we have:



$$\frac{AB}{500}=\frac{243}{300}, \text{ or } AB=\frac{243}{300}\times 500=405 \text{ yd., } \textit{Ans.}$$

2. To find the length of a pond or a swamp.
Stake off a line PC=150 yd. Measure the line CQ, which is found to be 100 yd. Extend PC to B, making CB=15 yd., and extend QC to A, making CA=10 yd. The triangles PQC and CAP



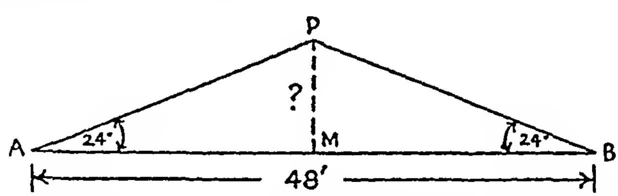
are therefore similar, since we deliberately made two pairs of sides proportional. Now, when AB is measured, it is found to be 21½ yd.

$$\text{Therefore } \frac{AB}{PQ} = \frac{15}{150}, \text{ or as } \frac{1}{10};$$

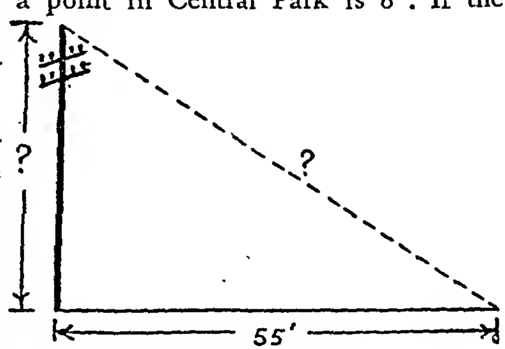
$$\text{or } \frac{21.5}{PQ} = \frac{1}{10};\ PQ = 215 \text{ yd., } \textit{Ans.}$$

PROBLEMS FOR PRACTICE: NO. 26

1. When the angle of elevation of the sun is 50°, what will be the length of the shadow (on the ground) of a pole 75 feet high?
2. When an airplane is directly over a certain point P, an observer at point Q, 450 yards from P, finds that the angle of elevation is 56°. How high was the plane?
3. The roof of a barn is 48 ft. wide. If the rafters make an angle of 24° with the horizontal, how high above the cross-beams is the ridgepole? (Find PM.)



4. From the top of the Empire State Building in New York, the angle of depression to a point in Central Park is 8°. If the Empire State Building is 1248 ft. high, how far away is observed point?
5. The guy wire supporting a telegraph pole makes an angle of 38° with the ground. If it is 55 ft. from the foot of the pole to the stake where the wire is fastened, how high is the pole? How long is the wire?



Areas of Similar Figures

We have seen that corresponding dimensions and lengths of corresponding sides of similar figures are in proportion. The *areas* of similar figures are in the ratio of the *squares* of their corresponding dimensions or sides. In other words, if two squares are 1″ and 2″ on a side, respectively, their areas are as 1 : 4, or,

$$\frac{\text{small square}}{\text{large square}}=\frac{1^2}{2^2}=\frac{1}{4}.$$

Or again, if two similar rectangles have dimensions of 2″×6″ and 3″×9″, their respective sides are in the ratio of 2 : 3, but their areas are in the ratio of $2^2 : 3^2$, or 4 : 9.

EXAMPLES

1. The floor dimensions of one room are 20′×30′, while those of another are 50′×75′. What is the ratio of their respective floor space?

   They are similar rectangles, with sides in the ratio of 20 : 50, or 2 : 5. Hence their areas are in the ratio of

   $2^2 : 5^2$, or 4 : 25, or 1 : 6¼, *Ans.*

2. Find the ratio of the areas of two triangles whose sides are 3, 5, 7 and 12, 20, 28.

   Since their sides are in the ratio of ¼, their areas are in the ratio of 1 : 16, *Ans.*

3. One of two squares is 16 times as large as the other. If the sides of the smaller square are each 2″ long, how long are the sides of the larger square?

   $$\frac{1}{16}=\frac{2^2}{n^2}, \text{ or } n^2=4\times 16$$

   $$n^2=64$$

   $$n=8, \textit{Ans.}$$

Circles may be regarded as similar figures; their circumferences are in the same ratio as their radii or their diameters.

In other words, if we double the radius of a circle, the circumference is doubled; if we divide the diameter by 4, the circumference is only $\frac{1}{4}$ as long. The areas of two circles are in the ratio of the *squares* of their diameters or radii. If we double the radius, the area is *four* times as large; if we triple it, the area is *nine* times as large; etc. If the areas of two circles are in the ratio of 1 : 2, their diameters are in the ratio of $1 : \sqrt{2}$, or 1 : 1.4; or, the diameter of the larger circle, which is twice as large in area, is only about 40% longer than that of the smaller circle.

EXAMPLES

1. The diameter of one apple pie is 8″, and that of another pie is 10″. How many times larger is the second pie than the first?

$$\frac{\text{area of 2nd pie}}{\text{area of 1st pie}} = \frac{10^2}{8^2} = \frac{100}{64} = \frac{25}{16},$$

or, 2nd pie is $1\frac{9}{16}$ times as large, *Ans.*

2. The diameters of two silver coins are 3 cm. and 4 cm., respectively. If the coins have the same thickness, how many times as much silver is there in the larger coin?

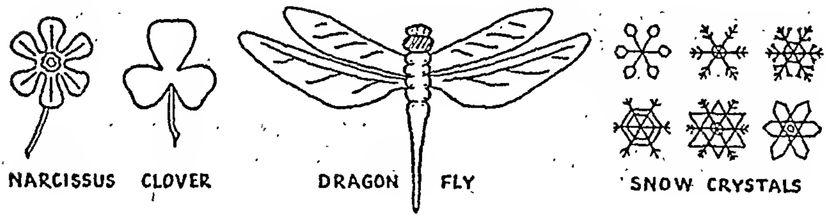
$$\frac{\text{area of large coin}}{\text{area of small coin}} = \frac{4^2}{3^2} = \frac{16}{9}$$

or, large coin is $1\frac{7}{9}$ times as large,
and, since they both have the same thickness, it contains $1\frac{7}{9}$ times as much metal, *Ans.*

SYMMETRIC FIGURES

Symmetry in Nature

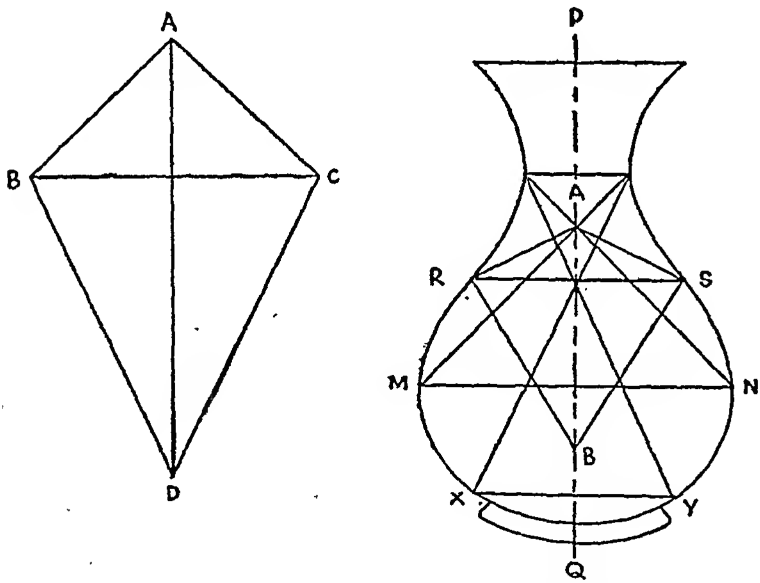
Many forms in Nature exhibit the geometric property of *symmetry*. Anything symmetrical, whether natural or man-made, is usually pleasing in appearance, since it is "balanced," and appeals to the eye. Symmetry is one of the most important principles of ornament, design and architecture. It is not, however, the only one; others are repetition and rhythm.



EXAMPLES OF SYMMETRY IN NATURE

Line Symmetry

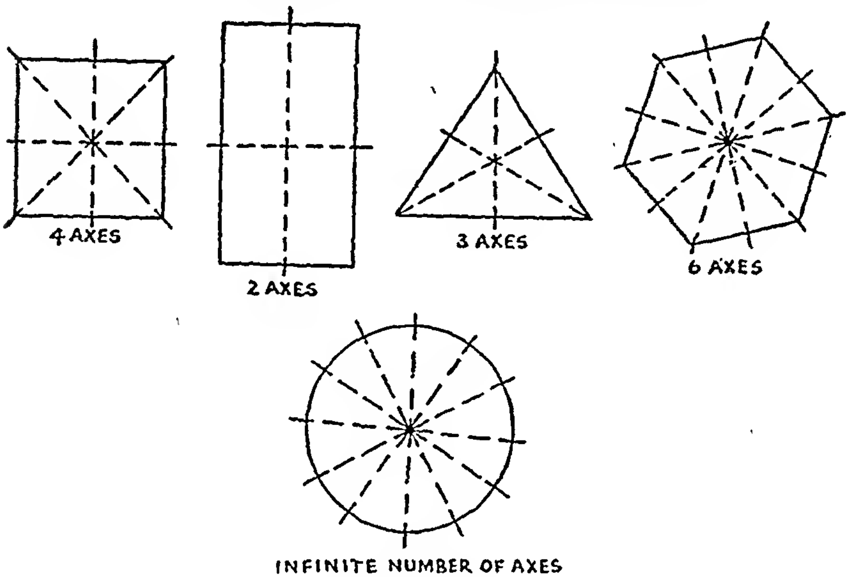
Geometric figures that are symmetric with respect to a line are said to have line symmetry. This line is said to be an *axis of symmetry*. If such a symmetric figure is "folded



over" along the axis of symmetry, the two halves will match exactly, or coincide. Thus if the kite figure is folded along AD, each half will fit on the other, but not if folded along BC. Hence AD is an axis of symmetry, while BC is not. In the vase figure, PQ is an axis of symmetry. Corresponding points, like R and S, M and N, etc., are equally distant from

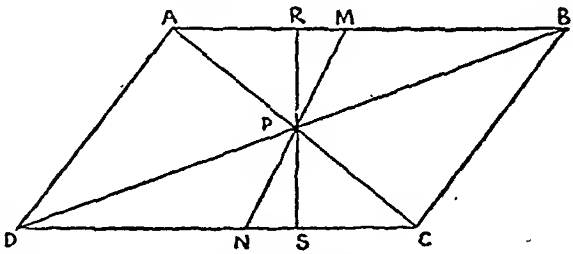
the axis; any point on the axis, like point A, is equally distant from corresponding pairs of points, like R and S, M and N, X and Y, etc.

Some figures have more than one axis of symmetry, as shown by the dotted lines in the accompanying illustrations.



Point Symmetry

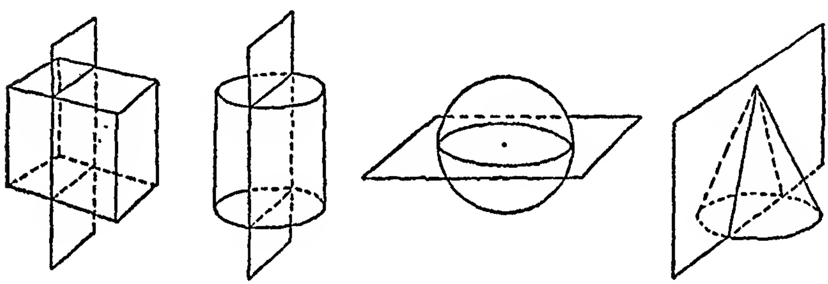
Consider the parallelogram, with its diagonals intersecting at P. It is evident that AP=PC, and DP=PB. It is also true that every other segment passing through P and terminating in the sides of the figure is also bisected at P; i.e., PM=PN, PR=PS, etc. In such a case the point P is called the *center of symmetry* of the figure. A cardboard pattern of the figure could be supported by a pin at its center of symmetry. Similarly, a square, a rectangle, any regular



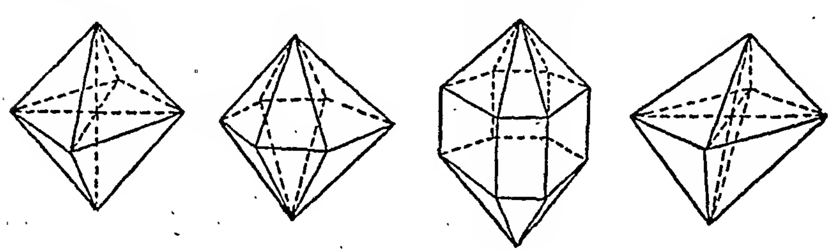
polygon with an even number of sides, and a circle, all have a center of symmetry; in the first three cases, it is the point of intersection of the diagonals; in the case of the circle, it is the center of the circle.

Plane Symmetry

When a solid figure can be divided by a plane surface so that each half is the counterpart of the other, the figure exhibits plane symmetry, and the dividing plane is called the plane symmetry. Corresponding points are equally distant from the plane of symmetry.



Many natural objects, including plants, animals and minerals, have one or more planes of symmetry. This is true also of the human body, a fact of the utmost importance in anatomy and physiology. The principle of plane symmetry is often employed or exemplified in the decoration of a shop window, the design of a house, the decoration of a table, the arrangement of furniture in a room, the construction of large buildings such as churches, theaters, auditoriums, etc.



CHAPTER VI

GRAPHIC METHODS

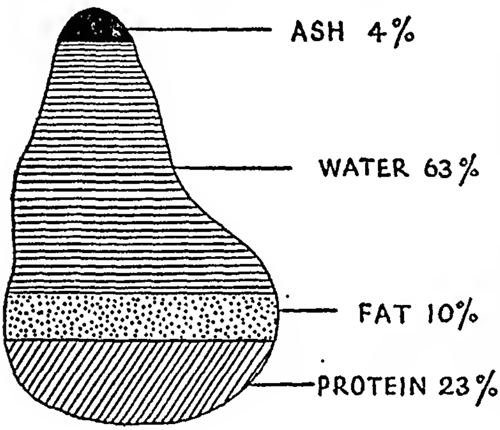
Over a century ago the scientist Von Humboldt claimed that "whatever relates to extent and quantity may be represented by geometrical figures. Statistical projections which speak to the senses without fatiguing the mind possess the advantage of fixing the attention on a great number of important facts." Hence the popularity and widespread use of graphs and charts. They are more direct, more forceful, and more dramatic than numerical statistics. No matter to what purpose—whether for study, for publicity, or for reference—graphs are, in general, very effective and economical.

You may expect to find graphs anywhere: in books, in periodicals, in newspapers, in pamphlets, on show cards in advertisements, in business reports, and so on. Their use, however, is sometimes limited. For one thing, they are of necessity less accurate than the figures on which they are based, which, of course, doesn't matter too much in many cases. In the second place, they are sometimes misleading, which may or may not be intentional. It is also possible that the reader of a chart or graph may misinterpret it. For these reasons, then, the present discussion will center on the typical graphic methods most likely to be encountered in general usage.

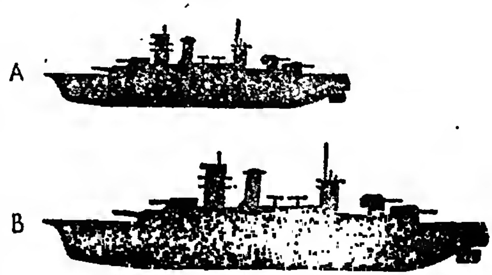
PICTORIAL GRAPHS

Pictographs

A type of picture-graph less commonly used than formerly is the pictorial representation of an object which has been arbitrarily subdivided to show certain numerical relationships; as, for example, the pictorial representation of the food values of beefsteak. This is a very poor type of graphic representation, and should definitely be avoided. The irregular outline of the picture as a whole, and of each of the shaded areas, makes a comparison of the areas difficult, if not altogether impossible; the shading only adds to the confusion.



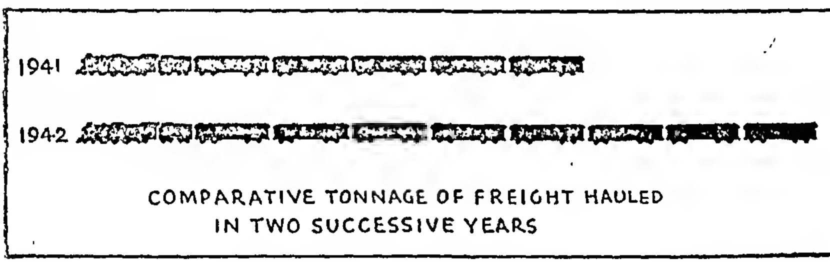
Another type of pictograph which cannot be approved mathematically is the device in which similar figures, outlined or shaded, are set next to each other for comparison, as shown in the silhouetted warships depicting the relative naval strengths of two nations A and B. Whether inked in or left in outline form, it should be remembered, as we learned in Chapter V, that the areas of two similar figures are to each other as the *squares of their linear dimensions*. Hence, whether intentionally or inadvertently, such comparisons may be quite misleading, even when accompanied by the corresponding numerical data, and even if attention is



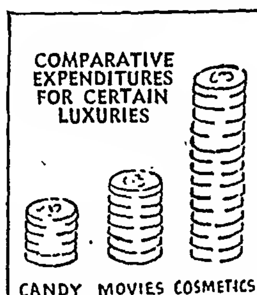
directed to the linear dimensions. For if the length of one ship is double that of the other, its area is *four* times that of the second; and if the area of one ship is twice that of the other, then its length is only approximately *1.4* times as long as that of the other (or only 40% longer). Now, whether we realize it or not, there is a strong tendency, psychologically, for the eye to prefer to compare *areas* rather than *distances*. It is clear, then, that such pictographs are apt to be quite untrustworthy.

Principle of Comparing Lengths

This latter type of pictograph has been gradually modified so that it embodies the principle of comparing lengths only. In the two illustrations, the trains of cars and stacks of coins to be compared are drawn to invite attention to their respective



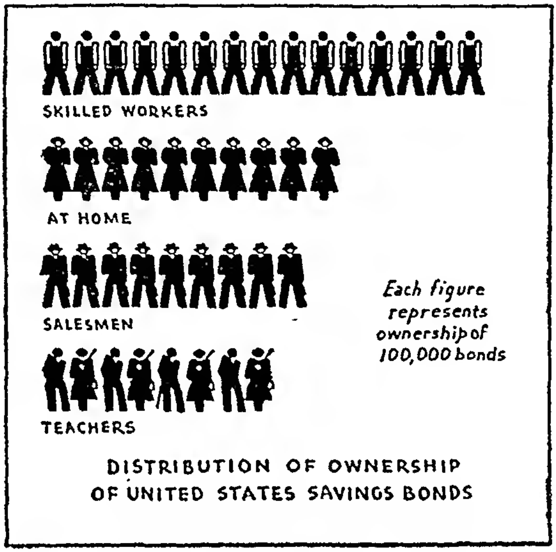
lengths and heights only; in short, linear extensions are compared and nothing else. This is both psychologically and mathematically much sounder and more reliable. Even so, if the bars of such a graph are unduly wide, heavily shaded, or of widely differing lengths, the comparison is not always easy to make accurately.



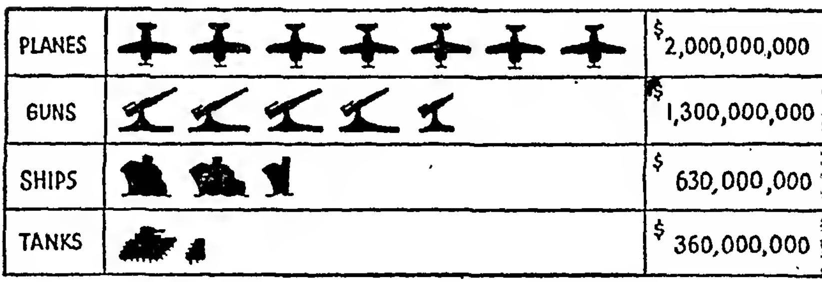
Isotypes

In recent years a still further modification known as the Neurath *isotype*, or Vienna picture method, has become

increasingly popular. It preserves the dramatic and psychological virtues without sacrificing mathematical correctness. Two typical illustrations of this device are shown herewith.



DISTRIBUTION OF OWNERSHIP OF UNITED STATES SAVINGS BONDS



RELATIVE DISTRIBUTION OF DEFENSE APPROPRIATIONS

A little careful study will show that the basic principle underlying these charts is the ratio of linear dimensions. This is the the same principle used in the bar graphs to be discussed shortly. The rows of symbols, when taken in at a glance, are in effect line segments whose lengths we are invited to compare visually. In good pictorial statistics the following rules should be observed:

(1) Only comparisons should be represented, not individual quantities.

(2) The symbols used should be simple, compact and self-explanatory.

(3) The pictorial chart should suggest only approximate quantities, not detailed amounts.

(4) For larger amounts, *more* symbols should be used, *not* larger symbols.

(5) The amount represented by each symbol should be clearly stated on the chart.

(6) The actual amount of each item depicted should also appear on the chart.

CHARTS AND GRAPHS

General Purposes of Graphs

In general, charts and graphs are intended to show one of three major things:

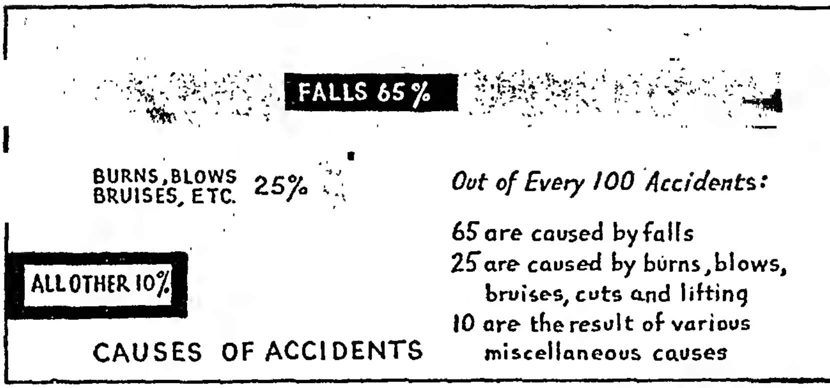
(1) the relation of the parts to the whole (100%-charts).

(2) a comparison of the amounts of different kinds of things, or of the same thing at two or more different times or places (Categorical charts).

(3) the rate of change, or historical trend, or the growth of something with the passing of time (Historical graphs).

100%-Charts

Two types of charts showing the relation of the parts to the whole are very commonly used—the 100%-bar chart, and the "circle graph" or "pie chart." Both of them are superior to the beefsteak pictograph on page 121, even though they lack pictorial qualities. Neither of them is necessarily expressed in terms of per cents, although they nearly always are; very often both the actual figures and the per cents are stated directly on the chart, which is usually desirable. A typical divided bar-



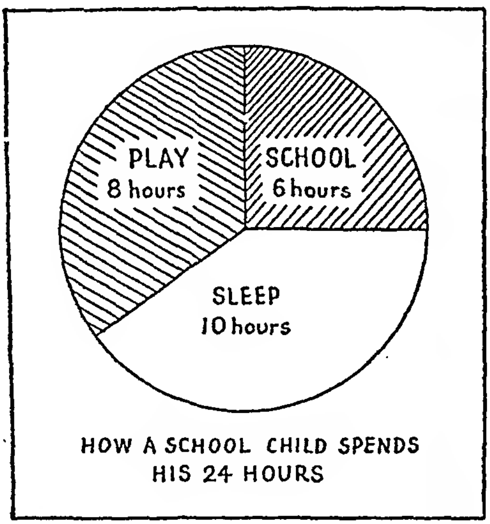
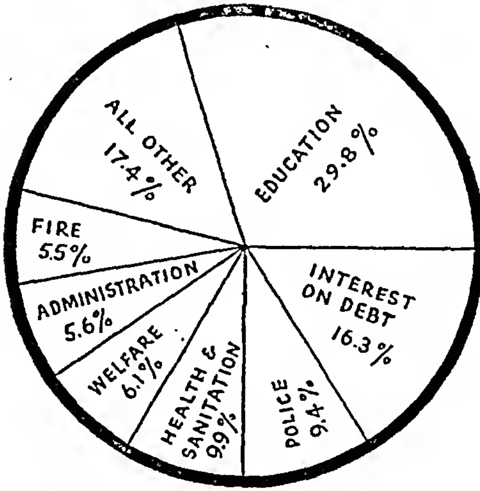


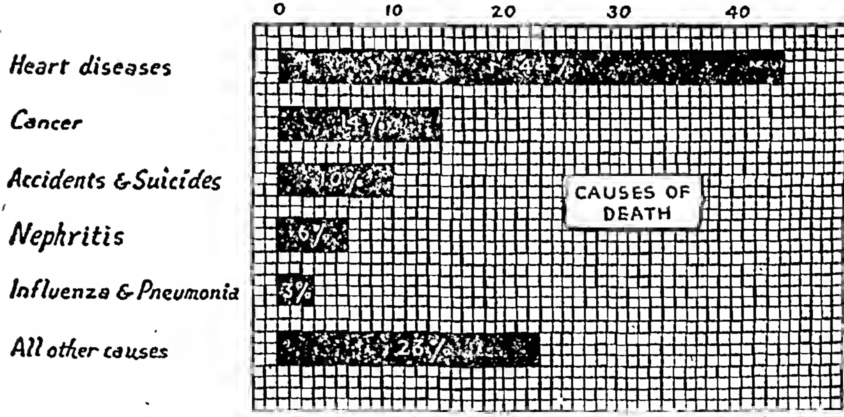
chart and a conventional divided-circle are shown here. Of the two, the divided bar-chart is perhaps slightly to be preferred, since here again, only lengths have to be compared by the reader, whereas a mental comparison of sectors in a circle having different central angles is not quite so simple.

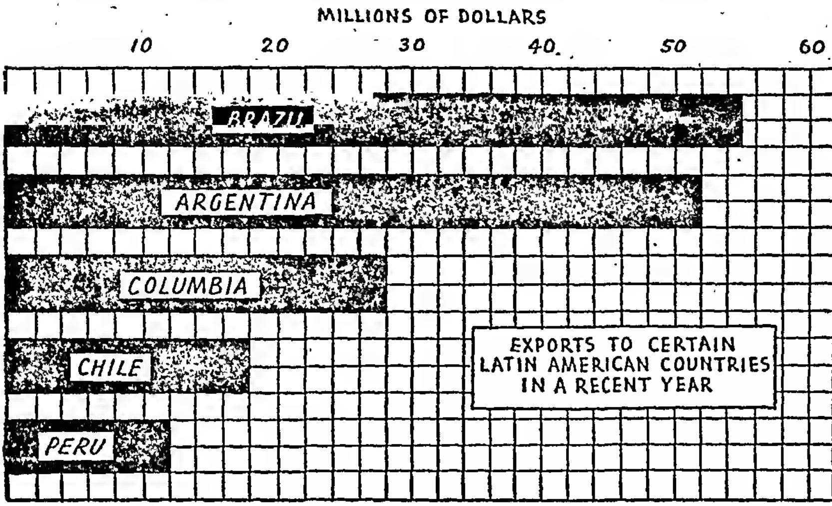


HOW NEW YORK CITY SPENDS ITS ANNUAL TAX MONEY IN A TYPICAL YEAR

Categorical Charts

The simplest, most direct, although perhaps least "interesting" method of making direct comparisons is by means of horizontal bar-charts, typical forms of which follow.





In these horizontal bar-charts, showing comparisons between different *kinds* of things, or between different *places,* only one numerical scale is required, viz., the scale representing the amounts involved. No other *numerical* scale is needed, since we are dealing with various *categories.* While not always the most effective device for exhibiting such comparisons, horizontal bar-charts are simple and convenient.

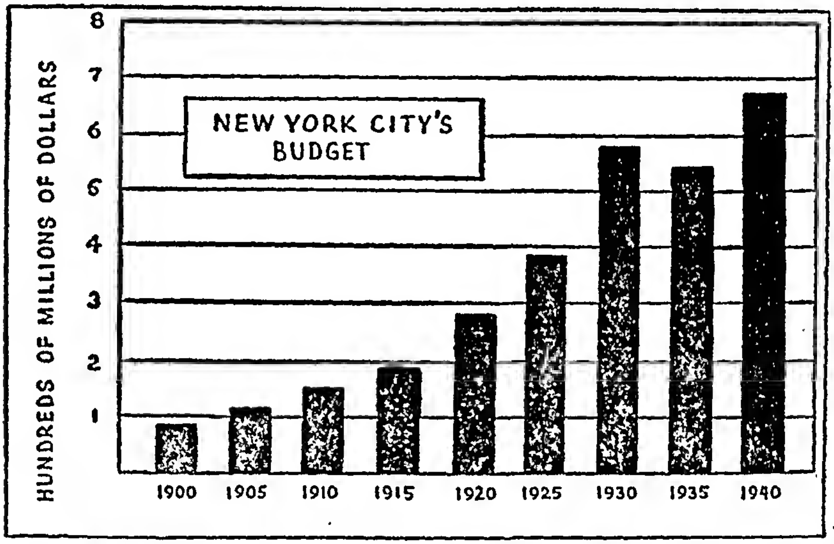
Historical Graphs

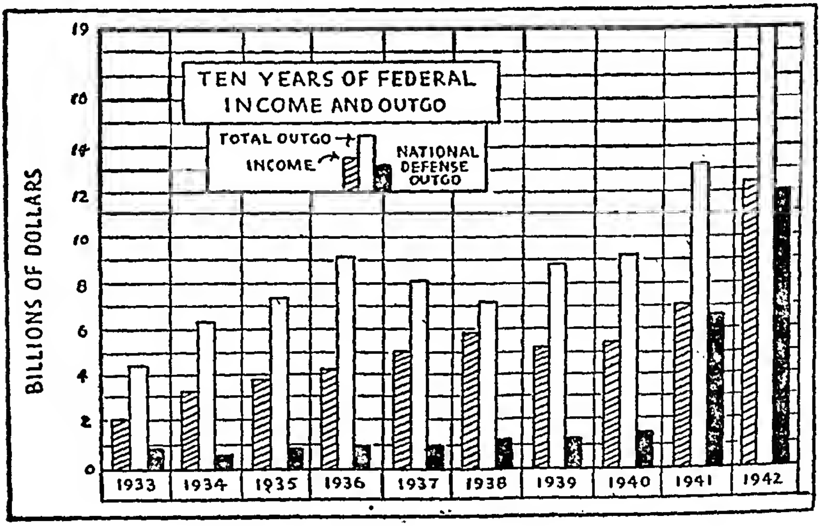
Graphs showing time changes, or the increases and decreases in the amount of something over a period of time, are generally of two kinds: (1) vertical bar graphs, and (2) broken- or smooth-line graphs. Both kinds differ from the categorical charts, as we shall see, in that they have *two scales* instead of only one; that is why it is preferable to call them *graphs* rather than *charts,* although these terms are used rather freely and interchangeably, and there is no standard convention.

Vertical Bar Graphs

Historical bar graphs should always be drawn with the bars placed *vertically,* not horizontally. The horizontal scale is always taken to represent the time, and the vertical scale repre-

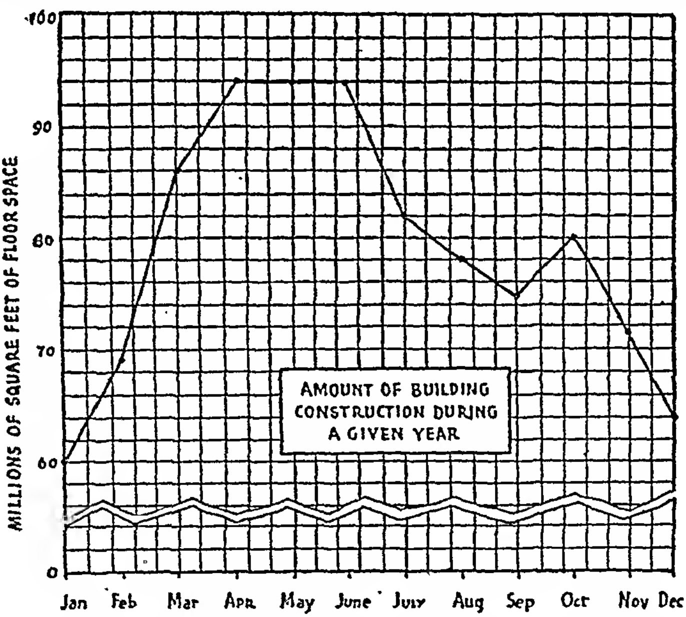
sents the quantities or amounts at various times. The "trend" may frequently be noted by inspection of the successive positions of the tops of the bars. Percentage increases or decreases from year to year (or from decade to decade) may also be approximated by comparing the heights of any two adjacent bars.





Broken-Line Graphs

A second method commonly used to represent time changes is the so-called *broken-line* graph, which also involves the use of *two* scales, as in the case of the vertical bar graph. The horizontal scale again always represents the time intervals; the vertical scale is used for the varying amounts. Several typical examples of broken-line graphs are given to illustrate the wide variety of data which may be exhibited advantageously by such graphs. The method of constructing broken-line graphs is simple enough: after selecting suitable scales, the points are "plotted" in accordance with the given figures; they are then joined by straight-line segments. This results in the so-called "broken-line" graph. Whether or not other values on the graph lying *between* plotted values are accurate, or have any meaning

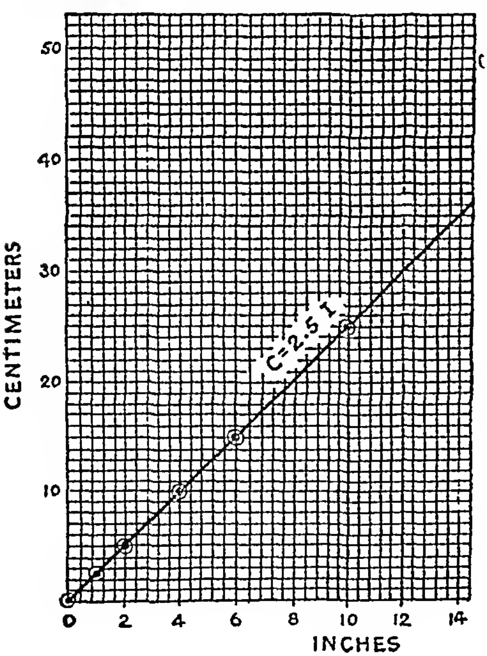


at all, depends entirely upon the nature of the quantities represented. If values on a broken-line graph between plotted points are used, it is implied that the successive values be-

Graph of a Linear Formula

The relationships expressed symbolically by formulas may also be represented graphically. Consider, for example, the formula C=2.5 I, showing the relation between inches and centimeters. If we substitute various numerical values for C, we obtain in each instance a corresponding value for I; these *pairs* of corresponding values may be arranged in a table as shown. These pairs of numbers may then be "plotted" as points on a graph, and then connected to one another in succession. It will be seen that the graph in this instance is a straight line. Such a relation between two variables is said to be a *linear* relation, or a straight-line function. Whenever two quantities are *directly proportional,* the graph of their relationship is a straight line. A table of values such as the above could be extended indefinitely, as there are countless (fractional) values between any two integral values of C (or of I); but the graph represents all of these at one time without the necessity of computing or tabulating them individually.

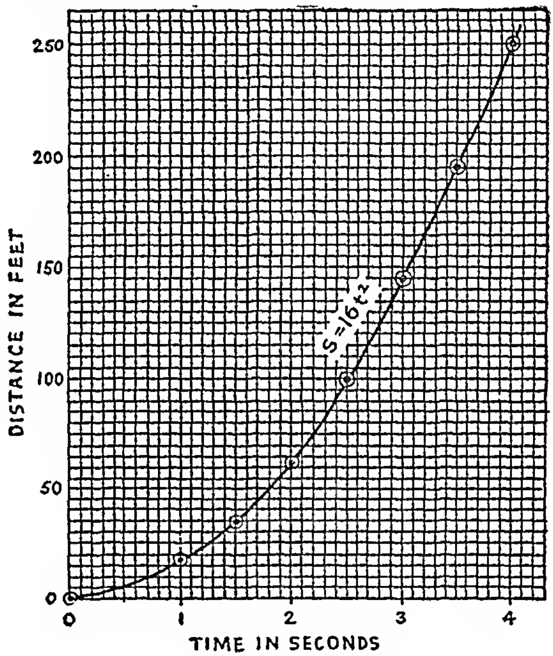
I	C
0	0
1	2.5
2	5.0
4	10.0
6	15.0
10	25.0
15	37.5
20	50.0



Graph of a Quadratic Formula

The graph of a quadratic formula turns out to be a curved line instead of a straight line. It is a special curve called a parabola, and is illustrated by the formula $s=16\ t^2$.

t	s
0	0
1	16
1½	36
2	64
2½	100
3	144
3½	196
4	256



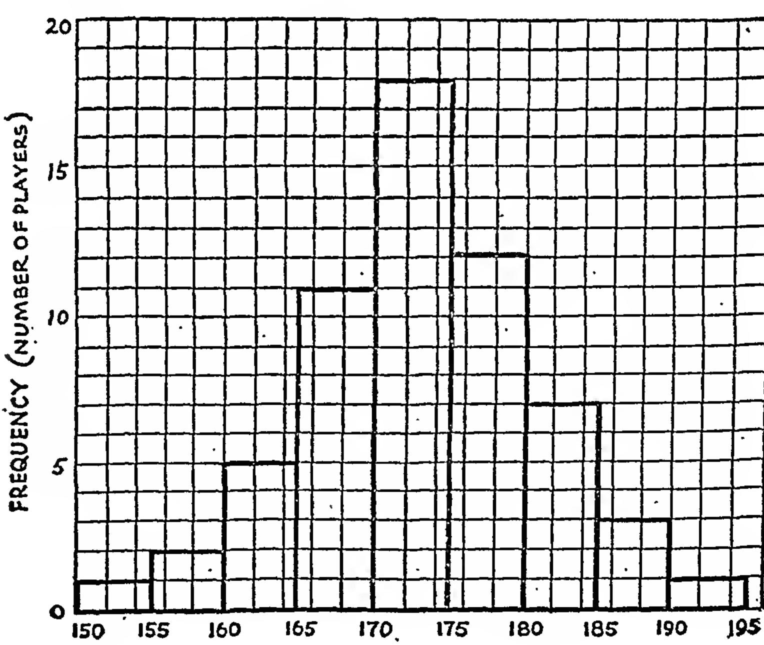
FREQUENCY DISTRIBUTION GRAPHS

Grouped Intervals

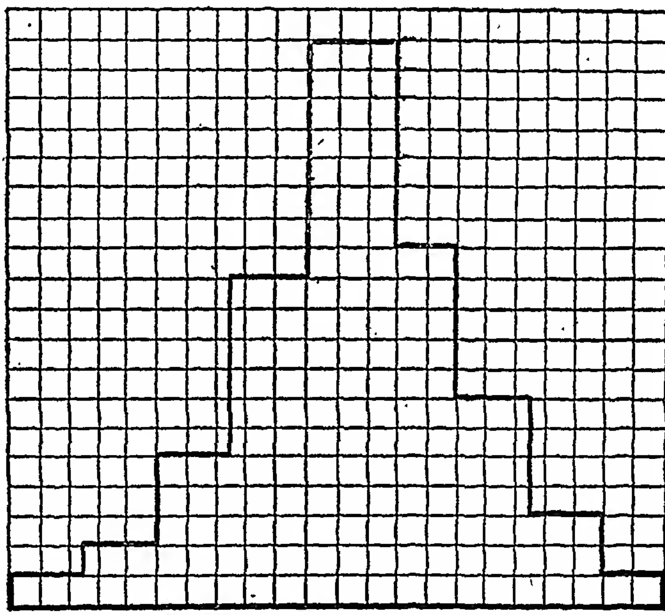
A rather different sort of statistical problem from any of the foregoing concerns the grouping of a considerable number of measurements or phenomena into convenient intervals. Consider the weights of a group of 60 players on a college football squad, which range, let us say, from 152 lb. to 191 lb. They might be grouped as shown in the table. Reading from the top down, this table means that of the 60 players, only 1 was 190 lb. or more, but less than 195 lb.; that 3 of the 60 were 185 lb. or more, but less than 190 lb.; that 7 of the 60 were 180 lb.

Interval of Weight	*Frequency of Occurrence*
190 – 194.9	1
185 – 189.9	3
180 – 184.9	7
175 – 179.9	12
170 – 174.9	18
165 – 169.9	11
160 – 164.9	5
155 – 159.9	2
150 – 154.9	1
TOTAL	60

or more, but less than 185 lb.; etc. The so-called "distribution of these frequencies" can be shown much more effectively by a *histogram* such as the following:



Such histograms are especially useful in showing at a glance the range of measurement (in the above instance, the weights) most predominant; the complete range between the extreme measurements; the prevalence (or not) of extremely large and small measurements; the tendency (or not) for the measurements to be distributed symmetrically on either side of a "central tendency"; and other significant features not readily inferred from the frequency table alone.



Histograms are sometimes drawn in outline only, or entirely "blacked in," in which cases they are sometimes referred to as "staircase diagrams."

PART III

Practical Uses of Mathematics

In everyday life we are continually making practical use of mathematics. The rest of this book deals with mathematics as applied to our everyday affairs. Thus Chapters VII–X deal with questions of ordinary household management, income and expenditures, and budgeting. Chapters XI–XV discuss the common problems arising in connection with owning one's home, the family car, travel and leisure. Chapters XVI–XVIII concern the affairs of business, including communication and transportation problems, office problems, and merchandising problems. Chapters XIX–XXIII deal with financial matters, such as borrowing money, installment buying, savings accounts, investments, life insurance and property insurance. The concluding Chapter XXIV deals with the mathematics of taxes. A careful study of Part III, therefore, should give the reader the ability to handle readily all the problems involving mathematics which are such an important part of modern life.

CHAPTER VII

MATHEMATICS OF THE HOUSEHOLD

SUCCESSFUL LIVING depends, among other things, upon good financial management. As Mr. Micawber says, in Dickens' *David Copperfield:* If expenditures exceed income, result—misery; if outgo is less than income, result—happiness.

The average family may be looked upon as a "going concern"; and in dealing with financial matters, the home should be conducted in accordance with sound business principles. The family income is generally known in advance, at least approximately; expenditures, made almost daily, can be in large part anticipated. To be sure, unforeseen contingencies are bound to arise—medical and hospital expenses, unanticipated assessments or taxes, price changes beyond the householder's control, etc. The sensible person, aware of this, accordingly provides for such possibilities. Successful management calls for planning.

In the present chapter we shall discuss a few of the general problems of personal expenditures and household expenses—how to buy wisely, how to stop the financial leaks—dealing in Chapter VIII with the more specific applications of arithmetic to problems of the kitchen, the cost of gas and electricity, the preparation of meals, and the importance of calories and diets.

BUYING WISELY

Buying in Quantity

Shopkeepers can usually afford to sell you goods at lower prices if you buy two or three or a dozen items of the same thing at one time. Buying in larger quantities whenever possible invariably leads to substantial savings. Many commonly used articles are sold in this way: thus a grocer offers canned goods selling for 13¢ each at 3 for 34¢; or a druggist offers toilet soap selling at 10¢ a cake for 97¢ a dozen; or handkerchiefs retailing at 29¢ apiece may be purchased at "4 for a dollar." The savings in such instances may seem trivial expressed in cents; but in terms of percentages their true significance is revealed. Moreover, it must be remembered that on staple commodities, i.e., items frequently and regularly purchased, a few cents on each occasion adds up over a longer period to a sizable number of dollars.

EXAMPLES

(1) 3 cans at 13¢ each = 39¢
3 cans for 34¢ = saving of 5¢
5¢ ÷ 39¢ = 12.8%, rate of saving

(2) 12 cakes of soap at 10¢ each = $1.20
12 cakes for 97¢ = saving of 23¢
23¢ ÷ $1.20 = 19.2%, rate of saving

(3) 4 handkerchiefs at 29¢ each = $1.16
4 handkerchiefs for $1 = saving of 16¢
16¢ ÷ $1.16 = 13.8%, rate of saving

Buying by Weight

Many items that were formerly sold by volume are now sold by weight, which is usually to the consumer's advantage, since variations in size, which affect the actual amount when measured by pecks, quarts, bushels, and boxes, or even by dozens,

are not so affected when measured by weight. Thus apples, potatoes, bananas and other fruit and vegetables are frequently offered at so much per pound. In purchasing small oranges at 29¢ a dozen, or large ones at 35¢ a dozen, the actual cost per pound may be very nearly the same. Eggs generally run ten to the pound, but at 34¢ a dozen you may be paying more *per pound* for small pullet eggs than you would by buying the regular size at 37¢ per dozen. There is only one way to be sure; check purchases and make comparisons by actually weighing, either in the store or at home. It pays to have a small scales at hand in the kitchen, with a capacity perhaps up to 10 pounds.

What Size Can?

The prejudice against canned foods, fortunately, is no longer as great as it once was. Food thrift can be further augmented by careful attention to the size of the jar or can, i.e., its capacity in ounces or fluid ounces, which is generally stated on the label. For example, a certain brand of preserves is offered at 17¢ for a 1-lb. jar, and 29¢ for a 2-lb. jar; a saving of 14.7%. Or again: a "small size" can of fruit juice is priced at 9¢, and the "large size" at 19¢. Which is the better buy? The respective labels reveal that the smaller can contains 1 pt. 2 oz., whereas the larger contains 1 qt. 14 oz. Let us see what this means:

1 pt. 2 oz.=16+ 2=18 oz.
1 qt. 14 oz.=32+14=46 oz.
At 9¢ for 18 oz. the rate=\$.005 an oz.
At 19¢ for 46 oz. the rate=\$.0041 an oz.
Difference per oz.=\$.0009

In other words, you are paying $9/41=22\%$ too much, or saving $9/50=18\%$, by buying the large can instead of the small can. Or you might say, if I buy the large can for but a trifle (1¢) more than twice the small one, and since twice 18 oz.=36 oz., I am getting about 8 oz., or half a pint gratis.

The sensible housewife will do well not only to become aware of the differences in the capacity of various-sized cans, but also to familiarize herself with the "standard" sizes. It should be pointed out further that on special sales of canned goods, "in-between-sized" or non-standard cans may be offered, and that the alleged saving may not be as great as it seems.

APPROXIMATE CONTENTS OF TINS OF CANNED GOODS

Can Size	*Weight*	*Contents in Cupfuls*
8 oz.	8 oz.	1
No. 1	11 to 14 oz.	1⅓
No. 2	18 to 20 oz.	2½
No. 2½	28 to 30 oz.	3½
No. 3	32 to 36 oz.	4
No. 10	104 to 108 oz.	13

Food Thrift

In other words, the purchasing of food is a factor of the budget where skill and constant alertness spell economy. A few cents saved here and there on a pound of butter, a can of tomato juice, or a dozen jars of preserves amounts to a considerable sum at the end of a month. Ten cents a day saved on food amounts to over $30 by the end of the year,—enough to pay the annual premium on a fair-sized life insurance policy. If the grocer's success depends upon small margins of pennies, because of a huge turnover, why shouldn't the smart, thrifty housewife take a leaf from his book?

EXAMPLES

1. Women's dress-shields are advertised at "4 pairs for 95¢; regularly 29¢ a pair." What is the per cent saved by purchasing four pairs?

$4 \times 29¢ = \$1.16$
$\quad\quad\quad\quad\;\; .95$
$\quad\quad\quad\;\; \$\ .21$, amount saved
$\$.21 \div \$1.16 = 18.1\%$, rate of saving

2. Vegetable soup is sold at 10¢ a can. It can also be purchased at 3 cans for 22¢. How much can be saved by purchasing a case of a dozen cans at the lower price? What per cent is saved? What is the saving per can?

(a) 1 doz. cans at 10¢ each $=$ \$1.20
1 doz. cans at 3 for 22¢ $=$ .88
\$ .32, amount saved on a dozen cans

(b) $\$.32 \div \$1.20 = 26.7\%$, rate of saving

(c) $30¢ - 22¢ = 8¢$, saving on 3 cans
$8¢ \div 3 = 2\frac{2}{3}¢$, saving on each can

3. If apples are sold at 10¢ a lb., or \$4.75 a bushel, what per cent is saved in buying by the bushel? Consider a bushel of apples to weigh 56 lbs.

$\$4.75 \div 56 = 8.5¢$ per lb.
$10¢ - 8.5¢ = 1.5¢$ saving per lb.
$1.5¢ \div 10¢ = 15\%$, rate of saving, *Ans.*

4. If the retail price of butter has advanced 35% over the price one year ago, what was the price last year when butter is now selling at 27¢ a lb.?

$135\% = \$.27$
$1\% = \$.27 \div 135 = \$.002$
$100\% = 100 \times \$.002 = \$.20$ per lb., *Ans.*

5. If the retail price of sugar should advance 22%, how much more per year would have to be paid for sugar by a family that uses an average of 20 lbs. of sugar per month and has been buying it at $5\frac{1}{2}¢$ a lb.?

$20 \times 12 \times \$.055 = \$13.12$
$\$13.12 \times .22 = \$2.89$, *Ans.*

Sales and Discounts

Many commodities, such as clothes, furniture, and household accessories are frequently available at "sale prices." Stores hold sales and offer discounts for a variety of reasons: taking inventory; change of style; out of season; odd sizes; shopworn merchandise; quick turnover; goods specially priced wholesale; and many others. In almost every instance, provided the sale is bona fide and the merchant's integrity is beyond question, the consumer stands to gain, sometimes quite considerably.

EXAMPLES

1. During a midsummer sale a fashionable dress shop offers a \$29.95 town dress for \$8.75. What per cent is saved if advantage is taken of this sale? How much is saved in buying two of these dresses?

$\$29.95 - \$8.75 = \$21.20$

$\$21.20 \div \$29.95 = 70.8\%$, rate of saving

$$\begin{array}{r} \$29.95 \\ 8.75 \\ \hline \$21.20 \\ \times 2 \\ \hline \$42.40 \end{array}$$

\$42.40, saved on 2 dresses

7. At a clearance sale a department store advertised 125 pairs of ruffled marquisette $60'' \times 80''$ curtains at \$1.98 a pair. If the original price was \$3.29 per pair, how much can Mrs. Crane save by buying 4 pairs at the sale price? What per cent is this?

$\$3.29 - \$1.98 = \$1.31$, saving per pair
$\$1.31 \times 4 = \$5.24$, saving on 4 pr.
$\$1.31 \div \$3.29 = 39.8\%$, rate of saving, *Ans.*

3. A 28-inch mahogany drum table that regularly sells for \$26.95 is sold for clearance at \$16.97; what per cent discount is this?

$\$26.95 - \$16.97 = \$9.98$, amt. of reduction
$\$\ 9.98 \div \$26.95 = 37.0\%$, rate of discount, *Ans.*

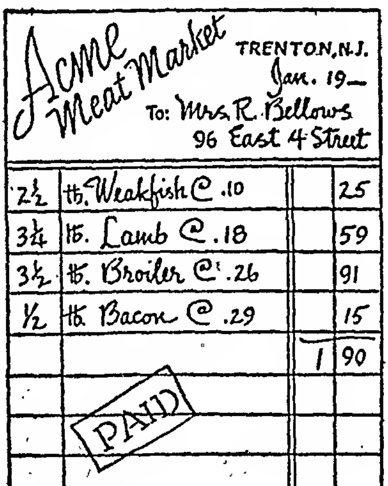
4. All the hats displayed in a milliner's shop are being sold at "40% off." If Mrs. Tracy selects a hat marked \$8.95, what will she have to pay for it now?

$\$8.95 \times .40 = \$3.58$
$\$8.95 - \$3.58 = \$5.37$, reduced price, *Ans.*

Consider, for example, a married couple with an income of \$3000 a year. If they allow 12% of their income for clothes, they plan to spend 12%×\$3000=\$360 a year for this item. Suppose they bought all their clothing at an average reduction in price of 20%. They could either buy 5/4 as much, or 25% more (5/4=1¼=125%=100%+25%) for the amount they had originally planned to spend. Or they could buy the same amount as originally contemplated at an actual saving of 20%×\$360=\$72, which could be used to pay the gas and electric bills for the entire year, or put into a savings account.

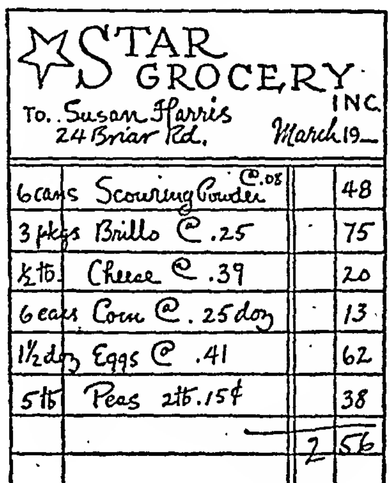
CHECKING YOUR BILLS

Neighborhood merchants, such as the grocer, the butcher, or the laundry, usually furnish the purchaser a sales slip or an itemized bill. Such bills should always be checked to see whether the entries have been made and extended correctly, whether the addition is correct, whether the items entered correspond to those received, and whether any credits or allowances due properly entered. The following are typical specimens of such bills. The amount of each item, multiplied by the unit price to obtain the total amount due, is called the extension.


Acme Meat Market TRENTON, N.J.
Jan. 19—
To: Mrs. R. Bellows
96 East 4 Street

2½	lb. Weakfish @ .10		25
3¼	lb. Lamb @ .18		59
3½	lb. Broiler @ .26		91
½	lb. Bacon @ .29		15
		1	90

PAID


STAR GROCERY INC.
To. Susan Harris
24 Briar Rd.
March 19—

6 cans	Scouring Powder @ .08		48
3 pkgs	Brillo @ .25		75
½ lb.	Cheese @ .39		20
6 ears	Corn @ .25 doz		13
1½ doz	Eggs @ .41		62
5 lb	Peas 2 lb. 15¢		38
		2	56

SUNSHINE
HAND LAUNDRY
MAIN ST. & CLINTON RD.
TOPEKA, KAN.

To: Mrs. T. Wilkinson
Alamo Place

5	Shirts .12		60
14	Handkerchiefs .02		28
8	Socks .05		40
4	Sheets .07		28
2	Pillow Cases .04		08
10	Towels .05		50
		2	14

Fred A. Bender
HARDWARE
& HOUSE FURNISHINGS
ATHENS, OHIO Oct 10—

To: Mr. John Haynes.
841 Hillside Ave.

5	qts. Turpentine @ .15		75
1/4	doz Hinges @ 2.60		65
3	gal. Paint @ 2.25	6	75
6	lbs. Linseed @ .85	5	10
		13	25

Gas Bills

The consumption of so-called illuminating gas, or gas for use in cooking, is measured in cubic feet. It is sold usually in accordance with different rates for various classes of service, residential, general, and large-volume consumers.

TYPICAL RESIDENTIAL RATE SCHEDULE

First	600 cu.ft. or less per mo.	....	$1.00
Next	2400 " " " " " "	....	.095 per 100 cu.ft.
Next	3000 " " " " " "	....	.07 " " " "
Next	4000 " " " " " "	....	.06 " " " "
All over	10,000 " " " " " "	....	.05 " " " "

MINIMUM CHARGE: One dollar per meter per month

The gas bill shown may be checked as follows:

9600 cu. ft.	600 cu. ft.	=$1.00
8200	800 cu. ft. at $.095 per C=	.76
1400 cu. ft. consumed		$1.76
		.04
		$1.80

TO THE VALLEY GAS COMPANY
HARRISBURG, PA.

SEE RATE SCHEDULE ON BACK OF BILL

TEL. MAIN 402

———— 19

Mary C. Rogers
Euclid Road
Townville, Pa.

OFFICE HOURS
9:00 A.M. 5:00 P.M.

RATE CLASS	READING DATES FROM	TO	METER READINGS PRESENT	PREVIOUS	CONSUMPTION 100 CU. FT.	AMOUNT CURRENT BILL	SALES TAX	ARREARS
A-1	MAR. 14	APR. 15	96	82	14	1.76	.04	—

TOTAL: 1.80

RECEIVED PAYMENT ——————————

Electric Bills

Electricity is consumed and sold in units known as kilowatt-hours. One thousand watts are equal to one kilowatt. It would require one kilowatt-hour to consume 1000 watts for one hour, to light a 100-watt lamp for 10 hours, a 25-watt lamp for 40 hours, or five 50-watt lamps for 4 hours, etc. The rates charged for electricity also vary with the class of service. Those shown here are typical residential rates.

MONTHLY RATE SCHEDULE: RESIDENTIAL ELECTRICITY

Energy Charge (exclusive of fuel adjustment)

For the first	10 kwh. (or less)	$.90	
For the next	35 kwh.	.05	per kwh.
" " "	40 kwh.	.04	" "
" " "	40 kwh.	.03	" "
For excess over	125 kwh.	.02	" "

MINIMUM CHARGE: $.90 per month (energy only)

FRANKLIN ELECTRIC LIGHT & POWER CO.
ROCHESTER, N.Y.
TEL. APPLE 2400

Mrs. Grace Kramer
18 Bushnell Drive
City

CUSTOMER'S RECORD OF PAYMENT
CHECK NO.
DATE

1941		CODE	METER READINGS		FUEL	SALES	ARREARS	CONSUMPTION	AMOUNT
FROM	TO		PREVIOUS	PRESENT	ADJUST	TAX		KWH.	
June 13	July 15	X	590	653	.02	.07	–	63	3.46

If arrears shown have already been paid, please disregard.

RATES ON REVERSE SIDE

This bill may be verified as follows:

653 kwh.	10 kwh.			=$ .90
590	35 "	@	.05=	1.75
63 kwh.	18 "	@	.04=	.72
				$3.37
				.02
				.07
				$3.46

Telephone Bills

Telephone service may be had under a variety of contracts. A typical residential "individual line" service is charged for as follows:

Monthly charge (including 66 free outgoing local calls) $4.25

Additional local calls up to 300 .... 5¢ each
next 300 " " .... 4½¢ "
next 300 " " .... 4¢ "
all over " " .... 3¾¢ "
Toll charges extra, according to zones.

EXAMPLE

During a certain month Mrs. Percy made 72 local calls; her out-of-town toll charges amounted to 65¢. What was the total amount of her bill?

Monthly service charge	$4.25
6 additional local calls @ $.05	.30
	$4.55
Local sales tax at 2%	.09
	$4.64
Toll charges	.65
Total:	$5.29

The sales tax at 2% should be found by dividing by 0.51, which gives the amount of the tax correct to the nearest cent.

Department Store Bills

Many women have charge accounts at their favorite department stores and receive from them monthly bills. Such bills should be carefully checked, not only as to the correctness of the entries, but also as to proper extensions, returns or credits, correct sales tax, and, of course, correct addition. By and large, billing clerks are quite accurate, but once in a while mistakes occur. If the sales slip is illegible, or there is a rush of business during the holiday season, you may be overcharged.

Stores sometimes allow a courtesy discount to professional people, such as teachers, nurses, and ministers. In computing the discount to which you are entitled, fractions of more than ½¢ *in the discount* may be taken as the next larger whole cent. The discount applies only to the amount purchased, excluding the sales tax; the tax is added to the net amount of the bill. The following will illustrate these points.

EXAMPLE

Mrs. Bailey's bill from the department store last month is shown. For what amount should she make her check when paying this bill, if she is entitled to a 6% discount?

A. H. Huntington, Inc.
Dayton, Ohio

To: Mrs. Ruth Bailey
220 Fernroad Ave.
Dayton, Ohio

Oct. 5	3 pr.	Gloves @ 2.98			8	94
		Sales Tax		18		
12	6 pr.	Stockings @ 3 for 3.59			7	18
		Sales Tax		14		
22	½ doz.	Towels @ 8.50 doz.			4	25
		Sales Tax		08		
22	1	Hat			6	95
		Sales Tax		14		
		Total Tax		54		
					27	32

\$27.32	\$27.32	\$25.68, net am't
.06	1.64	.54, tax
1.6392	\$25.68, net am't of bill excluding tax	\$26.22, am't of check to be sent
Discount=\$1.64		

STOPPING THE FINANCIAL LEAKS

Living within Your Income

Curious as it may seem, it is as difficult a task for a family with a \$10,000 a year income to live within that amount as it is for one with a \$2,000 a year income. Budgeting one's income is often very helpful (see Chap. VI). How often do we ask ourselves: Where *does* the money go? Where are the leaks? The examples that follow are simply suggestions and illustrations.

Incidental Expenses

Many of the little leakages have to do with incidental and sundry expenditures. For example: the Sunday paper may be delivered 52 times a year, at 5¢ extra; this comes to $2.60 annually. A trivial amount, perhaps; but it will pay for a year's subscription to your favorite magazine (if you live near enough to walk for the paper). Or again, suppose you smoke a package of cigarettes a day. In some shops you pay 15¢, in others 16¢ or even 17¢ a pack. At the end of a year, if you conscientiously never pay more than 15¢, you will have saved possibly in the neighborhood of $3.50 to $4.00; you might even extend this saving to $5.00 if you buy them by the carton or by the hundred. Five dollars a year will pay for your club dues, for a couple of books you have wanted, or for a handbag for your wife. You may argue that unless you have a tin box, and put pennies into it like a child, you'll never see that $5.00. You may be right, but the fact remains that you have *actually spent $5.00 less for cigarettes than you might have,* and that if you haven't the money, then you have *spent it for something else,* even though you may be unaware of it. Or again, suppose that during the course of a summer a young woman has her white shoes cleaned at 25¢ each cleaning. With two pairs of shoes requiring in all an average of three cleanings a week, over a period of three months (13 weeks), it will cost her 3×13×25¢, or $9.75 for the season. With 75¢ worth of cleaning fluid, she could do them herself and save $9.00.

Similarly, there are scores of ways in which annoying leaks can be stopped, i.e., turning electric lights off when not needed; temporarily disconnecting the telephone when away in the summer; buying a 14 oz. bottle of mouth wash for 59¢, instead of a 1½ oz. bottle for 10¢; keeping the refrigerator properly regulated; turning the water faucet and hose off when not in use; and so on. There is genuine satisfaction in stopping these leaks, provided we don't go to extremes.

Buying for Service

Some of the commodities we buy are quickly consumed or comparatively short-lived. Others are purchased from the standpoint of service. Wearing quality in many cases is of as much importance as initial cost. This is true, for example, of upholstered furniture, rugs, linens, and, to a lesser extent, of clothing. Each has its own special considerations.

Sheets and Pillowcases

Very little, if anything, is saved in buying sheeting by the yard and hemming your own sheets. At the very most, you may save 10¢ a sheet which hardly pays for your trouble. The wise woman will know precisely what size sheet she needs; if in doubt, she will measure the size and thickness of the mattress, and allow generously for tucking in. For general use, sheets should be 99″ or 108″ long, although sheets 90″ and less are to be found in the shops. Satisfactory widths are as follows:

Single or twin bed	63″
Twin or three-quarter bed	72″
Double bed	81″ or 90″

Sheets bought at bargain sales are often smaller than standard size.

When repeatedly laundered, sheets change in size. Continued ironing in one direction increases the measurement in that direction and decreases it in the other. Sheets ironed from selvage to selvage may increase 3% or 4% in width, and decrease 7% or 8% in length; part of this decrease is due, however, to shrinkage. Sheets are hardly ever preshrunk. For this reason they should be bought a little larger than necessary, because it costs less to buy them a few inches longer than to pay for the extra cost of preshrinking. Manufacturers sometimes give figures on the number of times a sheet can withstand laundering. A good quality sheet can withstand from 200 to 250 launderings before it is no longer serviceable. Such figures are

an approximate index of durability. Thus a sheet laundered on an average of six times a month would last a little over three years. Pillowcases should always be bought a little larger than the pillow: e.g.,

Size of Pillow	*Size of Case*
21″×27″	42″×36″ or, 42″×38½″
22″×28″	45″×36″ or, 45″×38½″

Blankets

Most manufacturers of blankets abide by an agreement as follows:

1. If labeled "All Wool" a blanket contains 98% or more wool.
2. If labeled "Part Wool" it contains from 5% to 25% wool.
3. If it contains more than 25% (but less than 98%) of wool the label must state the minimum guaranteed per cent of wool.
4. If it contains less than 5% of wool, the word "wool" must not appear on the label.

In general, the greater the proportion of wool, the warmer the blanket will be.

Blankets should be purchased large enough to permit generous tucking in at the foot of the bed, comfortably reaching over the person's shoulders, and be wide enough to hang down over the sides of the mattress. To estimate the length of blanket to buy, add the length and thickness of the mattress, and add not less than 6 inches to allow for tucking in. To estimate the width, measure the width of the mattress and add the depth of the two sides. This gives the minimum size; it is always better to allow a few inches each way for shrinkage and for take-up by the body. The following table of standard sizes may also prove helpful:

For Single Beds (Inches)	*For Twin Beds (Inches)*	*For Double Beds (Inches)*
54×76	66×76	70×80
60×76	66×80	72×84
60×80	66×84	80×90
60×84	66×90	

Laundry Expense

This is a constant item of expense which permits, however, of considerable latitude. One can, for example, send *all* one's laundry to the commercial establishment; or one can send some of it, and do the rest at home; or one can do all, or nearly all of it, at home. A "hand" laundry will charge typical rates as follows:

A. *In Bulk:*

(a) All flat work (no clothing, curtains, etc.; except shirts):

First 10 lbs.	$1.00
Each additional pound	.08
Each shirt	.06

(b) Flat work and clothing, etc.:

First 10 lbs.	$1.25
Each additional pound	.10
Each shirt	.06

B. *Itemized:*

(Each type has its own net rate)

Shirts	.12	Bath Towels	.05
Handkerchiefs	.02	Tablecloths	.10
Socks, per pr.	.05	Napkins	.03
Shorts	.07	Sheets	.07
Pajamas	.15	Pillowcases	.04
Kitchen Towels	.04	Washcloths	.03

Let us suppose Mrs. Saunders' laundry for a certain week includes the following items:

6 shirts	$ .72
12 handkerchiefs	.24
10 pr. socks	.50
8 pr. shorts	.56
2 pr. pajamas	.30
6 kitchen towels	.24
4 bath towels	.20
1 tablecloth	.10
6 napkins	.18
4 sheets	.28
2 pillowcases	.08
5 washcloths	.15
Total wt.=21 pounds	$3.55

At the itemized rates, this would come to $3.55. If, however, it were done according to schedule A(b), the cost would be:

First 10 lbs.	$1.25
Next 11 lbs. @ 10¢	1.10
6 shirts @ 6¢ extra	.36
Total	$2.71

Now, if Mrs. Saunders were to do the socks, shorts and pajamas in an electric washing machine in her apartment or her house, and send the shirts together with all the flat work (now only 19 lb.), she would avail herself of schedule A (a) rates, and the cost would be:

First 10 lbs.	$1.00
Next 9 lbs. @ 8¢	.72
6 shirts @ 6¢ extra	.36
	$2.08
Use of electric washing machine	.10
	$2.18

Finally, if she lived in a house equipped with a home laundry, she could do the entire laundry at an operating cost (washing machine, dryer, and ironing machine) for water, electricity and

gas, for probably not more than 15¢. Naturally, the initial cost of such laundry equipment must be prorated (spread over) a number of years; for the sake of approximate concrete illustration, suppose equipment originally costing $300 has a useful life of 10 years; the cost is then $30 a year. If it is used 60 times a year, the cost is 50¢ each time it is used; or 65¢, with operating expenses included. Compared with an average weekly laundry bill of $2.50, the annual bill is $130, as compared to a home laundry cost of possibly $39:

Prorated cost	$30
Operating expense (60×$.15)	9
	$39

In short, doing it at home is only about ⅓ as costly. And even if help is hired to do the work, the saving is still very considerable.

Making Things Yourself

The thrifty woman, if she is so inclined, can save many a dollar by making things herself. Of course it takes time, it's work, and one has to be skillful with needle and thread. But let's look at the figures. Mrs. Cutler decides to make her daughter Joan, 13 years old, a school dress. Here is the cost of materials:

Dress pattern	$ .20
2½ yd. Dimity print @ 45¢	1.13
1¼ yd. Rickrack trim @ 12¢	.15
½ doz. Buttons @ 89¢	.45
Total Cost	$1.93

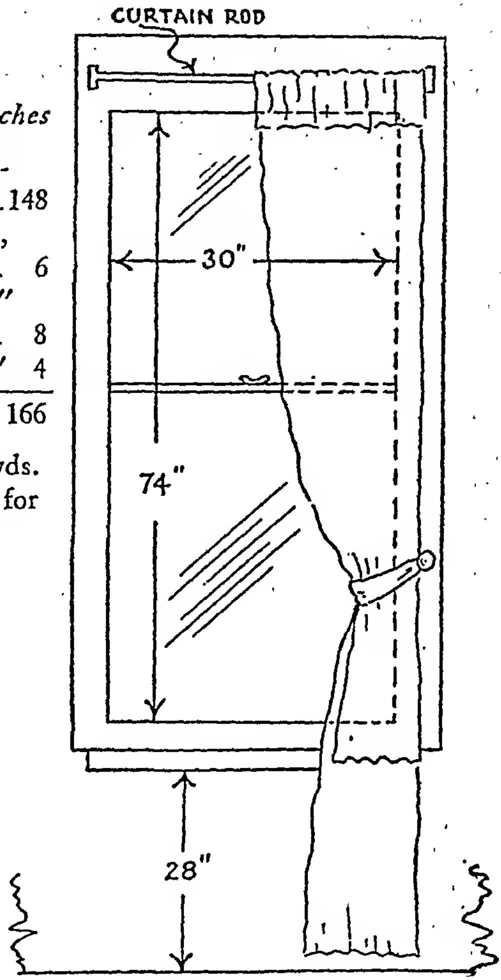
A similar ready-made dress might cost $3.89; saving: $1.96, or slightly over 50%.

Or perhaps she needs 4 pairs of simple glass curtains. Here is how she would calculate the material needed for one pair:

Window height=74″
Window width=30″

For 100% fullness:	*Inches*
Two lengths 36″-wide material	148
Double hem on bottom, ½″ deep, 3″ each	6
Heading and casing (4″ each)	8
Shrinkage, not less than 2″	4
	166

166″÷36″=4.61; hence 4¾ yds. of material will be required for each pair of curtains.



A simple rule to follow is to add 9″ to the exact window height, and then double that amount. (For "apron" length or full floor-length side draperies, the calculation is similar, except that 10″ might be added because of the tie-back effect; also special adjustments depending upon valence, linings, etc.) Now Mrs. Cutler, who needs 4¾ yds. of material for each of her four pairs of glass curtains, will have to buy 19 yds. of material; Dotted Swiss at 49¢ a yd. would cost her 19×\$.49=\$9.31. Similar curtains ready-made might cost \$3.98 a pair, or \$15.92. If she makes them she will save \$6.61, or almost 42%.

CHAPTER VIII

MATHEMATICS OF THE KITCHEN

WE COME NOW to the more specific problems of the kitchen, that necessary and delightful province where the lady of the house reigns supreme. In grandfather's day, more likely than not, coal and kerosene featured prominently in the kitchen; today these have been all but completely supplanted by gas and electricity. Nor was it likely that grandfather heard much about calories, vitamins, and balanced diets. Yet today they are commonplace topics of discussion, and are being given increased attention by more and more people all the time.

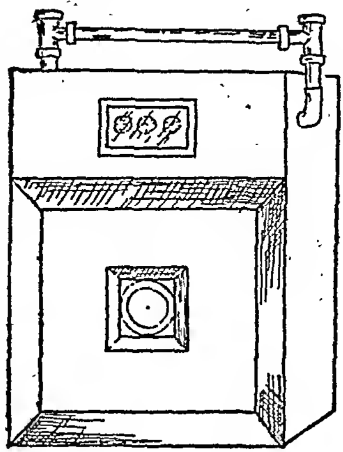
In this chapter, therefore, we shall discuss briefly the mathematics of gas and electric consumption, of household appliances, of recipes and formulas, and of calories and diets and menus.

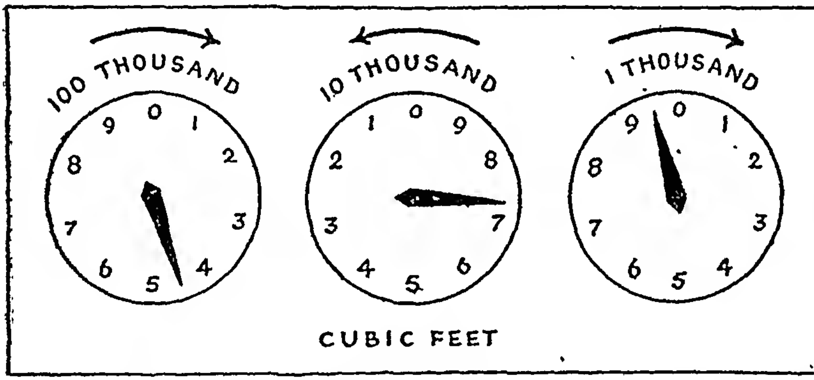
GAS AND ELECTRICITY

The Gas Meter

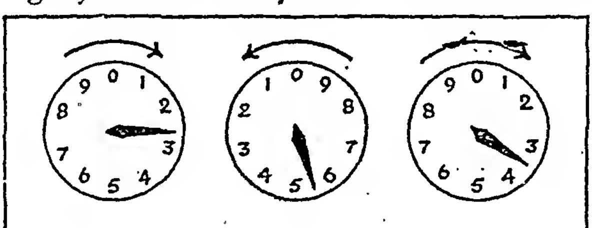
As we saw in the previous chapter, the gas used in the kitchen range is measured in cubic feet. It is measured as it passes through a gas meter, sometimes located in the kitchen, though usually in the cellar. The number of cubic feet that

have passed through the meter is read on three dials, such as shown in the drawing. The dial at the right reads in hundreds of cubic feet; the second dial, thousands of cubic feet; the last on the left, tens of thousands. In reading the meter, always read the *figure that the needle has just passed;* remember that the first and third needles move clockwise, and the middle one, counterclockwise. The reading on the dials shown herewith is 47,900 cu. ft.





In checking your bills, you should also check the meter readings, making allowances for the fact that several days or a week may have elapsed between the time the gas man has read the meter and the time you receive the bill. Let us illustrate once more. You get your bill on May 8. You consult the dials on your

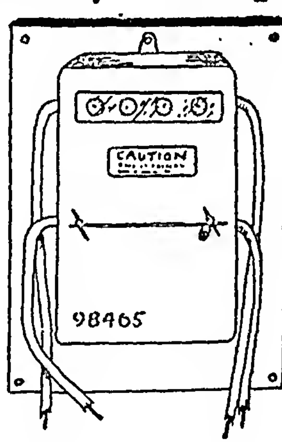


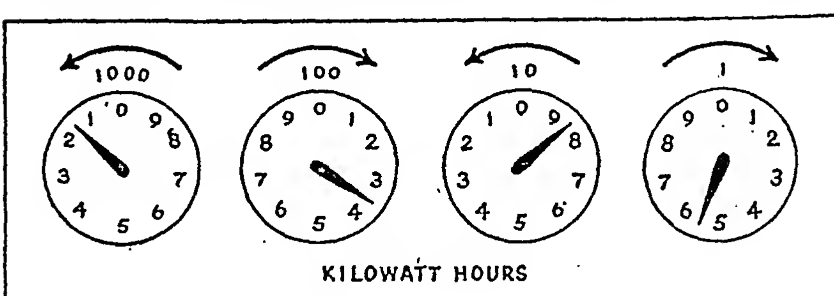
meter, which stand as shown; you read this correctly as 25,300 cu. ft. You consult your bill, which says "Present Reading: 252." Since this means the number of 100 cu. ft., it is really 25,200 cu. ft., which is doubtless correct. The "Previous Reading" (which you checked similarly last month) says "243," or 24,300 cu. ft. Thus in the month just passed you have consumed 900 cu. ft. Consulting the rate schedule (see page 146), you check the bill as before:

600 cu. ft.	\$1.00
300 cu. ft. @ 9½¢ per C	.29
Total	\$1.29

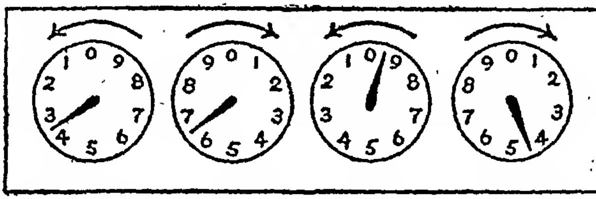
The Electric Meter

Electric meter dials are read in a manner very similar to gas meter dials. Electricity is measured in units of kilowatt-hours (kwh.). Just as the gas meter measures the number of cubic feet of gas consumed, so the electric meter indicates the number of kilowatt-hours of electrical energy consumed. An electric meter generally has four dials. The first dial on the right reads kwh. in units, the next in tens, the third in hundreds, and the last (at the left) in thousands. The dials shown below show a reading of 1385 kwh.





Electric bills may be checked the same way. You receive the bill on Jan. 10. The dials on your meter are set as shown, which



you interpret correctly as 3694 kwh. Upon consulting the bill, the "present reading" is given as 3686, which you assume then is correct. The "previous reading" is given as 3562, which means that you have consumed 124 kwh. since Dec. 10 or thereabouts. Using the rate schedule given on page 147, you check the bill as follows:

First 10 kwh.	\$ .90
Next 35 kwh. @ .05	1.75
Next 40 kwh. @ .04	1.60
Next 39 kwh. @ .03	1.17
Total	\$5.42

Cost of Using Gas

Ordinarily, gas is used in the average household for cooking. It is sometimes used in instantaneous hot water heaters, in mechanical refrigerators, and is coming to be used more and more for heating the entire house, instead of coal or oil. Let us look briefly into the matter of costs.

The top burner on an ordinary range, when turned up to a medium height, consumes from 25 to 35 cubic feet of gas per hour. If turned on full (so that you hear a gentle "roar"), it may be consuming gas at the rate of from 50 to 60 cubic feet per hour. However, you may not be getting twice the amount of heat, because proportionately more heat is lost from a large flame than from a medium flame. It is therefore poor company to use too large a flame. Oven burners vary, but consume more than a "top" burner—perhaps 60 to 80 cubic feet per hour, de-

pending upon how wide the valve is open, and at what temperature the oven is set.

EXAMPLES

1. The burners on the top of Mrs. Lang's kitchen range consume 27 cu. ft. of gas per hr. when turned fairly low; the oven, set for a roast, burns 75 cu. ft. per hr. If the roast is in the oven for 2 hours and 20 minutes, and she uses all four top burners for 45 minutes for the rest of the dinner, what is the cost of gas consumed in preparing the meal, if the rate is 94¢ per 1000 cu. ft.?

   2 hr. 20 min. $= 2\frac{1}{3}$ hr.
   $75 \times 2\frac{1}{3} = 175$ cu. ft.
   $27 \times \frac{3}{4} \times 4 = 81$ cu. ft.
   256 cu. ft.
   $\$.94 \times .256 = \$.24064 = 24$¢, *Ans.*

2. A large automatic hot water heater consumes 100 cu. ft. of gas per hour. If it is used on the average of ½ hour each day, what is the cost of operating it for one month (30 da.) at 90¢ per 1000 cu. ft.?

   $100 \times \frac{1}{2} \times 30 \times \frac{1}{1000} = 1.5$ thousand cubic feet
   $1.5 \times \$.90 = \$1.35$, *Ans.*

Cost of Operating Electrical Appliances

Today, in most sections of the country, electric lights, toasters, vacuum cleaners, fans, etc., are fairly common; in other parts of the country their use is being increased, due in large measure to extensive electrification as a result of Federal power projects. Electrical appliances consume energy in varying amounts for the same length of operation. The amount used per hour is called the "rating," and is generally stamped on the appliance. Thus an electric light bulb rated at 60 watts will consume, when in use, 60 watts every hour; an electric iron, rated at 600 watts, will consume 600 watts per hour, or ten times as much as the bulb. Some average ratings for common household appliances are as follows:

Electric kitchen clock	1	watt
Electric light bulbs	15–100	watts
Electric fan (10″)	50	"
Small portable radio	75	"
Electric mixing (beating) machine	100	"
Electric refrigerator	125	"
Electric clothes-washing machine	200	"
Vacuum cleaner	250	"
Electric coffee percolator	400	"
Electric clothes iron	600	"
Electric toaster	600–700	"
Electric waffle iron	700	"
Electric heater	800	"
Electric stove (2-burner)	1000	"
Electric laundry ironing machine	1200	"

EXAMPLES

1. An electric toaster is rated at 660 watts. If it is used 10 minutes a day, what is the cost of electricity consumed in a month (30 da.) at 8½¢ per kwh.?

   $660 \times 1/6 \times 30 \times 1/1000 \times \$.085 = \$.28$ a month, or about 1¢ a day

2. Mrs. Carrol's vacuum cleaner consumes 240 watts per hour. If she uses it 15 minutes a day on an average of 8 times a month, what is the monthly cost at 8¾¢ per kwh.?

   $$240 \times 1/4 \times 8 \times \frac{1}{1000} \times \frac{\$.35}{4} = 4.2¢ \text{ a month}$$

3. If a family uses, on the average, eight 40-watt lamps for 4 hours each night during the winter season, what is the monthly cost of lighting at 9¢ per kwh.?

   $$\frac{8 \times 40 \times 4 \times 30}{1000} \times \$.09 = \$3.46 \text{ per month}$$

4. The electric wall clock in Mrs. Jenkins' kitchen operates on 1 watt. How much does it cost for electricity to operate the clock for one year, if electricity costs 8½¢ per kwh.?

   $365 \times 24 \times 1 \times 1/1000 \times \$.085 = \$.7446$, or 75¢ a year

Further Points about Household Use of Electricity

The reader should note, by studying the table of average ratings of appliances, that there is a considerable difference between appliances that operate by producing heat, such as toasters or flatirons, and those which provide mechanical energy, such as an electric mixer or fan. Consider the following comparison:

> A radio consuming 75 watts is switched on continuously for 4 hours. How does the cost of running the radio compare with that of operating an electric flatiron rated at 600 watts for ½ hour?
>
> Radio: $(75 \times 4) \div 1000 = .3$ kwh.
>
> Flatiron: $(600 \times \frac{1}{2}) \div 1000 = .3$ kwh.

There is the same consumption of power, but one is operated *eight times as long* as the other. So that in equal periods of time, the one consumes 8 times as much power, and therefore costs 8 times as much to operate, regardless of the electric power rate. Of course, this 8:1 ratio could be seen at once from their respective ratings: 75:600=1:8.

It is also worth noting that some appliances are rated in *amperes* instead of watts. An ampere is a unit of electric *quantity,* not a unit of *energy*. Energy is the product of quantity (i.e., amperes) times volts, which gives *watts*. Thus,

$$\text{Amperes} \times \text{Volts} = \text{Watts};$$

and a flatiron rated at 5 amperes operating on 110 volts would use

$$5 \times 110 = 550 \text{ watts.}$$

Power means energy used per unit of time; hence *electric power* consumption is expressed as *energy required* times *time energy is used;* or as *watts* $\times$ *hours*. Thus 1 watt of energy used for 1 hour is equivalent to 1 watt-hour; 1000 watts used for 1 hour is equivalent to 1000 watt-hours, or 1 kilowatt-hour; 200 watts used for 3 hours is equivalent to 600 watt-hours, or 0.6 kilowatt-hours; etc.

Sometimes an appliance using an electric motor is rated as so-many horsepower; 1 horsepower (H.P.)=about 746 watts.

EXAMPLES

1. An electric heater is rated at 7½ amperes. If it is operated 6 hours a day on a 120-volt circuit, what is the daily cost of operation at 9¢ a kwh.?

   $7.5 \times 120 \times 6 \times 1/1000 \times \$.09 = \$.486$, or nearly 50¢ a day

2. An electric laundry machine is driven by ¼ H.P. motor. How much will it cost to operate it per hour if electricity costs 8½¢ per kwh.?

   $746 \times 1/4 \times 1 \times 1/1000 \times \$.085 = \$.01585$, or about 1.6¢ an hour

Every housewife has noticed that the electric bills are smaller in the summer than in the winter, because longer days mean less use of artificial lighting. For example, a monthly electric bill might vary as follows:

Jan.	$5.30	May	$3.90	Sept.	$3.30
Feb.	5.12	June	3.48	Oct.	3.94
Mar.	4.56	July	3.12	Nov.	4.46
Apr.	4.08	Aug.	2.96	Dec.	5.08

The average monthly charge for the above yearly record is $4.11.

Some apartment houses include the cost of electric refrigeration in the rental.

3. The refrigerator in Mrs. Randolphs' apartment operates on 100-watts. What is the monthly cost of operating it at 8¢ per kwh., running 28 days a month, since she defrosts it overnight once a week? Assume also that the thermostatic control causes it to be operated only half the time.

   $100 \times 24 \times 28 \times 1/2 \times 1/1000 \times \$.08 = \$2.69$

RECIPES AND FORMULAS

Measuring in the Kitchen

When cooking and baking, it pays to measure instead of guessing. Some of the commonly used measures and equivalents are given here for convenience:

1 pint	= 2 cups	1 tablespoon	= 3 teaspoons
1 gill	= ½ cup	1 cup	= 16 tablespoons
1 (liq.) oz.	= 2 tablespoons	1 glass	= 1 cup

4 cups flour	= 1 lb.
2 cups sugar	= 1 lb.
2 cups butter	= 1 lb.
3 cups meal	= 1 lb.
2 cups solid meat	= 1 lb.

EXAMPLES

1. How many oz. of butter in one tablespoon?

   16 tablespoons = 1 cup = ½ lb. = 8 oz.
   1 tablespoon = $\frac{8}{16}$ = ½ oz., *Ans.*

2. What is the weight in oz. of 6 tablespoons of flour?

   16 tablespoons = 1 cup = ¼ lb. = 4 oz.
   1 tablespoon = $\frac{4}{16}$ = ¼ oz.
   6 tablespoons = $6 \times \frac{1}{4} = 1\frac{1}{2}$ oz., *Ans.*

3. How much do 8¼ cups of sugar weigh?

   2 cups sugar = 1 lb.
   1 cup = ½ lb.
   8¼ cups = $8\frac{1}{4} \times \frac{1}{2} = \frac{33}{8} = 4\frac{1}{8}$ lb., or 4 lb. 2 oz., *Ans.*

4. Four teaspoons of milk is equivalent to how many liquid ounces?

   2 tablespoons = 1 (liq.) oz. = 6 teaspoons
   1 teaspoon = ⅙ oz.
   4 teaspoons = $4 \times \frac{1}{6} = \frac{2}{3}$ (liq.) oz., *Ans.*

Changing a Recipe to Make More or Less

In using recipes or formulas, the amounts of the various ingredients frequently have to be changed to make a larger or smaller number of portions. This is readily done by a simple use of ratios.

EXAMPLES

The following recipe for "Tomato Surprise" will make 4 portions. Mrs. Jenkins wishes to serve 12 people; how should she change it? How should she change it to serve 6 people?

For 4 servings	For 12	For 6
4 tomatoes	12	6
1 cucumber	3	1½
½ cup celery	1½ cups	¾ cup
2 chopped eggs	6	3
⅔ cup chopped ham	2 cups	1 cup
1½ tbsp. chopped onions	4½ tbsp.	2¼ tbsp.
½ cup mayonnaise	1½ cups	¾ cup

Making a Weaker Solution

It is often desired to dilute a given solution to some given strength. For example, suppose we wish to dilute 8 oz. of a 50% solution of acetic acid (vinegar) to a strength of 20%; how much water should be added? Let us say *x* oz. of water are necessary; then

$$\frac{\text{no. of oz. of acid}}{\text{total no. of oz. of liquid}}=\frac{4}{8+x}=\frac{20}{100};$$

$$\text{or,}\quad \frac{4}{8+x}=\frac{1}{5}$$

$$8+x=20$$

$$x=12,\ \textit{Ans.}$$

Or again, suppose 6 oz. of 95% alcohol are to be diluted to 40%; adding x oz. of water to the 6 oz. mixture gives

$$\frac{\text{no. of oz. of alcohol}}{\text{total no. of oz. of liquid}}=\frac{(.95)(6)}{6+x}=.4;$$

$$\text{or,}\quad .4x+2.4=5.7$$

$$.4x=3.3$$

$$x=8\tfrac{1}{4}\text{ oz., } \textit{Ans.}$$

Other Mixture Problems

Many other practical problems with mixtures are likely to arise in the kitchen or in the shop.

EXAMPLES

1. If the radiator of a car contains 10 qts. of a solution that contains 15% anti-freeze compound, and 2 qts. of water are added, what per cent of anti-freeze will the radiator contain? (Radiator capacity is 12 qts.)

$$\frac{\text{no. of qts. of ingredient}}{\text{total no. of qts. of liquid}}=\frac{(10)(.15)}{10+2}=\frac{x}{100}$$

$$12x=150$$

$$x=12\tfrac{1}{2}\text{, or } 12\tfrac{1}{2}\%\text{, } \textit{Ans.}$$

2. If 20 cc. of a 25% acid solution are added to 10 cc. of 100% acid solution, what per cent of acid will the resulting mixture be?

$$\frac{\text{no. of cc. of acid}}{\text{total no. of cc. of liquid}}=\frac{\tfrac{1}{4}(20)+10}{30}=\frac{x}{100}$$

$$\frac{15}{30}=\frac{x}{100}\text{, or } x=50\text{, or } 50\%\text{, } \textit{Ans.}$$

3. A pharmacist added a certain amount of water to 30 oz. of a 50% solution of argyrol, making the resulting solution 10%. How much water did he add?

$$\frac{\tfrac{1}{2}(30)}{30+x}=\frac{1}{10}$$

$$30+x=150$$

$$x=120\text{ oz., } \textit{Ans.}$$

4. If 3 pints of a 40% solution are mixed with 5 pints of a 20% solution, what is the per cent of the resulting mixture?

$$\frac{3(.4)+5(.2)}{8}=\frac{x}{100}$$

$$8x=220$$

$$x=27\tfrac{1}{2}\%\text{, } \textit{Ans.}$$

CALORIES AND DIETS

Food Nutrients

All our foods contain, among other things, three important nutrients: (1) *proteins,* (2) *fats,* and (3) *carbohydrates.* The proteins help to build our body tissue and muscles, and the fats and carbohydrates furnish heat and energy for our bodies. Table I shows how the proportions of these nutrients vary in different foods. These percentages do not add up to 100% for each food; the difference is due to the presence of other items, such as vitamins, mineral matter and water.

TABLE I

Food	*Protein*	*Fats*	*Carbo-hydrates*
Milk	3.5%	3.7%	4.9%
Eggs	13.0	12.0	...
Butter	1.0	85.0	...
Bread	9.0	2.0	54.0
Potatoes	2.0	...	18.0
Ham (smoked)	20.0	21.0	...
Beefsteak	23.0	10.0	...
Carrots	1.5	...	7.0
Beans, lima	18.0	2.0	66.0

EXAMPLES

1. How many ounces of fat are obtainable from ¼ lb. of smoked ham?

   16 oz.=1 lb.
   ¼×16=4 oz.
   4 oz.×.21=.84 oz., *Ans.*

2. An average glass of milk is equivalent to half a pint. If a pint of milk weighs approximately one pound, how many ounces of protein does a glass of milk contain?

   1 glass=½ lb.=8 oz.
   8 oz.×.035=.28 oz., *Ans.*

3. (a) What is the ratio of carbohydrate content in 1 lb. of bread to that of 1 lb. of potatoes?

(b) A 12-oz. loaf of bread is cut into 12 slices; an average potato weighs 5½ oz. What is the comparative carbohydrate yield of a sandwich (2 slices) and an average potato?

(a) $54\% \div 18\% = 3:1$, *Ans.*

(b) $12 \text{ oz.} \times 2/12 \times .54 = 1.08$ oz. carbohydrates in two slices of bread

$5\frac{1}{2} \text{ oz.} \times .18 = .99$ oz. carbohydrates in an average potato

$1.08 \div .99 = 1.09$; or 9% more in the bread

or, $1.08 - .99 = .09$

$.09 \div .99 = 1/11 = .09$, or 9% more in the bread, *Ans.*

4. "Pure" proteins and carbohydrates both average 1860 calories per lb.; "pure" fats and oils, on the other hand, average 4220 calories per lb. What is the energy value ratio of proteins or carbohydrates to fats and oils?

$1860 \div 4220 = .441$, or 44.1%, *Ans.*

or, expressed in another way:

$4220 \div 1860 = 2.27$, or, roughly, fats and oils yield about 2¼ times as much heat energy as proteins or carbohydrates, pound for pound.

Calories Available per Pound

The energy supplied by foods is measured in terms of the amount of heat they furnish when chemically changed in the body by digestion. The amount of heat is expressed in units called calories. A *calorie* is the amount of heat required to raise the temperature of one gram of water through one degree Centigrade. Table II gives the approximate energy value of various common foods, both in terms of the number of calories available per pound of food, and also in terms of the number of calories furnished by an average portion of food.

TABLE II. TABLE OF FOOD VALUES

Approximate No. of Calories per pound	*Food*	*Calories per portion*	
		Size of Portion	*Calories*
	FRUITS		
210	Apples	1 (medium)	80
290	Bananas	1 (medium)	100
160	Grapefruit	½ aver. grapefruit	90
165	Oranges	1 (medium)	90
155	Peaches	1 (medium)	35
700	Pineapple	1 aver. slice	100
	DAIRY PRODUCTS		
3500	Butter	1 tablespoon	100
2000	Cheese (American)	1½ cu. in.	100
1950 (pt.)	Cream (heavy)	1 tablespoon	60
700	Eggs	1 (medium)	70
330 (pt.)	Milk (whole)	1 glass	165
180 (pt.)	Milk (skim)	1 glass	90
	BREAD, CAKE, ETC.		
1200	Bread (white)	aver. slice	45
2400	Bread (whole wheat)	aver. slice	100
3000	Cake (layer)	aver. slice	250
800	Ice Cream	½ cup	200
1900	Graham Crackers	1 cracker	40
1870	Soda Crackers	1 cracker	25
	VEGETABLES		
190	Beans (string)	½ cup	20
140	Cabbage (chopped raw)	½ cup	12
160	Carrots (cubed)	½ cup	30
90	Lettuce	¼ head, small	10
350	Peas	½ cup	65
300	Potatoes (white)	1 (medium)	100
450	Potatoes (sweet)	1 (medium)	200
110	Spinach	½ cup	20
100	Tomatoes	1 medium	40

Approximate No. of Calories per pound	*Food*	*Calories per portion*	
		Size of Portion	*Calories*
	MEATS & FISH		
1500	Beef (rib roast)	1 aver. slice	100
1360	Ham (boiled)	1 aver. slice	100
1600	Lamb Chop (lean)	1 aver. chop	100
870	Pork Chop (lean)	1 aver. chop	200
460	Halibut	1 med. piece	150
880	Salmon	1 cup	220

EXAMPLES

1. How many times as many calories does one get from peas as from beans, assuming equal servings?

   $65 \div 20 = 3.25$, or $3\frac{1}{4}$ times as many.

2. What is the total caloric-value of a luncheon plate consisting of $\frac{1}{8}$ lb. of boiled ham, $1\frac{1}{2}$ medium white potatoes, and $\frac{2}{3}$ cup of carrots?

   $\frac{1}{8} \times 1360 = 170$ calories, ham
   $1\frac{1}{2} \times 100 = 150$ calories, white potatoes
   $\frac{2}{3} \times 60 = \underline{40}$ calories, carrots
   360 calories, total

3. Small pullet eggs average 12 to the pound. How many calories are there in a breakfast of 2 slices of toast (white bread), $\frac{1}{2}$ slice of ham, and 2 boiled eggs?

   Toast .... $2 \times 45 =$ 90 calories
   Ham .... $\frac{1}{2} \times 100 =$ 50 calories
   Eggs .... $\frac{2}{12} \times 700 =$ $\underline{117}$ calories
   Total 257 calories

Comparative Cost of Calories in Different Foods

If we compare the calorie-content of various foods and the difference in price, it is clear that some foods, considering the energy value they furnish, are cheaper than others. Sugar, for example, since it is practically 100% carbohydrates, is one of the

cheapest foods, in this sense. Bread and cereals in general are also relatively cheap. Eggs, on the other hand, are comparatively expensive. Meats, varying considerably in price, have to be compared separately. Fish is seemingly cheap, but because of the low percentage of calories per pound, it is apt to be relatively expensive.

EXAMPLES

1. What is the cost of 1000 calories of halibut at 28¢ a lb.? of ribs of beef, at 30¢ a lb.?

$\frac{28¢}{460}\times 1000=\$.61$ for halibut, per 1000 calories available

$\frac{30¢}{1500}\times 1000=\$.20$ for beef, per 1000 calories available

Or, on this basis, halibut is about three times as expensive as beef.

2. Find the comparative cost, on the basis of 1000 calories yield each, of a 16-oz. loaf of white bread at 11¢, and white potatoes at 4½¢ a lb.

$\frac{\text{Price per lb.}}{\text{Calories per lb.}}\times 1000=\text{cost per 1000 calories}$

Bread: $\frac{1\times \$.11}{1200}\times 1000=\$.09$ per 1000 calories

Potatoes: $\frac{\$.045}{300}\times 1000=\$.15$ per 1000 calories

$\$.15\div\$.09=1\frac{2}{3}$; or, potatoes are approximately 66⅔% more expensive than white bread on the basis of calorie-value.

How Much Food per 100 Calories?

For determining the calorie-value of a meal, and particularly for arranging balanced and special diets, you may find it helpful to refer to a chart (see Table III) showing the amounts or sizes of portions of various foods for every 100 calories.

TABLE III. FOOD PORTIONS PER 100 CALORIES

Food	*Approximate Amount per 100 Calories*
FRUITS	
Apple	1 large apple
Banana	1 medium banana
Grapefruit	½ large grapefruit
Orange	1 large orange
Orange juice	1 cup
Pineapple juice	⅔ cup
DAIRY PRODUCTS	
Butter	1 tablespoon (Pat 1″×1″×⅓″)
Cheese (American)	Cube 1⅛″×1⅛″×1⅛″
Cream (heavy)	1 ⅔ tablespoons
Eggs	1 large egg
Milk (whole)	⅝ cup
Milk (skim)	1⅛ cup
BREAD, CAKE, ETC.	
Bread (white)	2 slices (½″ thick)
Bread (whole wheat)	1 slice (½″ thick)
Cake (layer)	½ small slice
Ice Cream	¼ cup
Graham Crackers	2½ crackers
Soda Crackers	4 crackers
Sugar	5 teaspoons
VEGETABLES	
Beans (string)	2⅓ cups (cut)
Cabbage (raw)	4 cups (chopped)
Carrots	1⅔ cups (cubed)
Lettuce	2 large heads
Peas	¾ cup
Potatoes (white)	1 medium (baked potato)
Potatoes (sweet)	½ medium potato
Spinach	2½ cups (chopped)
Tomatoes	2 large tomatoes

Food	*Approximate Amount per 100 Calories*
MEATS AND FISH	
Beef (roast)	1 average slice
Ham (boiled)	1 average slice
Lamb Chop (lean)	1 average chop
Pork Chop (lean)	½ average chop
Tuna fish	½ cup
Salmon	½ cup

EXAMPLES

Find the total calorie-content of each of the following meals:

BREAKFAST

Orange Juice	½ cup	50	calories
Coffee	1 cup	0	"
Sugar	2 tsp.	40	"
Cream	1 tbsp.	60	"
Toast (white bread)	2 slices	100	"
Eggs	1 lg. egg	100	"
Ham	½ slice	50	"
	TOTAL	400	calories

LUNCH

Pineapple Juice	½ cup	75	calories
Lettuce	⅛ head	6	"
Salmon	½ cup	100	"
Peas	⅓ cup	44	"
Potatoes (white)	1 medium	100	"
Milk	1 glass	165	"
Cake	½ sm. slice	100	"
	TOTAL	590	calories

How Many Calories Are Needed?

The answer to this question depends upon one's age, size, and kind of work. For example, a man working in an office all

day, and taking little or no exercise, might not require more than 2500 calories a day; while a husky, hard-working laborer might need 5000 calories a day or more. The following tables* suggest the average energy requirements of boys, girls, and adults.

TABLE IV

ENERGY REQUIREMENTS OF CHILDREN

Age	*Calories Per Day*	
	Boys	*Girls*
Under 1 year	300– 900	300– 900
1–2 yrs.	900–1200	900–1200
3–5 yrs.	1200–1500	1200–1500
5–9 yrs.	1500–2000	1500–2000
10–13 yrs.	2200–3000	1800–2400
14–17 yrs.	2500–4000	2200–2600

The physical activity of work or play has an important effect upon the energy requirements of a person.

TABLE V

ENERGY REQUIREMENTS OF ADULTS from Age 20 to Age 60, in Calories per Hour per Pound of Body Weight

ACTIVITY	Calories per hr. per lb. of body weight
Sleeping	0.4
Slight exercise (sewing)	0.6
Light exercise (walking)	1.0
Active exercise (housework)	2.0
Severe exercise (chopping wood)	3.0

*These two tables are quoted from *Lennes* Practical Mathematics, by permission of The Macmillan Company, publishers.

Elderly people require less energy value from their food than do the young and middle-aged. To determine the number of calories required by a person over 60, calculate his needs as an adult according to Table V, and then reduce the result as follows:

Age 60 to 69	10%
Age 70 to 79	20%
Age 80 and over	30%

EXAMPLES

1. Mrs. Emory, who weighs 118 lb., sleeps 8 hours a day, does 4 hours of housework a day, and 12 hours slight exercise the rest of the day. How many calories does she require?

   8×0.4= 3.2 calories per lb. while sleeping
   4×2.0= 8.0 " " " " doing housework
   12×0.6= 7.2 " " " for rest of day
   18.4 " " "

   118 lb.×18.4=2,171 calories per day, *Ans.*

2. John Mitchell, a machinist's helper in a factory, weighs 178 lb.; he does heavy work 8 hours a day, slight exercise 9 hours a day, and sleeps 7 hours a day. What is his daily calorie requirement?

   8×3.0=24.0
   9×0.6= 5.4
   7×0.4= 2.8
   32.2, total calories per day per lb. of body weight
   178 lb.×32.2=5,732 calories per day, *Ans.*

3. Grandfather Stockton, who is 76 years old and weighs 145 lb., sleeps 10 hours a day, and does only slight exercise while awake. How many calories a day does he require?

   10×0.4=4.0
   14×0.6=8.4
   12.4 calories per day per lb.
   145 lb.×12.4=1,798 calories, if under age 60
   1798×20%=360
   1798—360=1,438 calories per day, *Ans.*

Special Diets

People who are overweight frequently change to a reduced diet. This means "low-calorie" menus, although very special diets should generally be undertaken under the advice of a physician if the overweight condition is serious. This is particularly true of "high-caloric" menus planned for persons below average weight.

EXAMPLES

The following are a typical day's menus for a person on a low-calorie diet:

Breakfast

	calories
1 orange	100
1 egg (boiled)	70
1 slice whole wheat toast	50
1 cup coffee	0
1 tsp. sugar	20
1 tbsp. heavy cream	60
Total	300

Lunch

	calories
½ grapefruit	100
1 slice white bread	50
1 portion vegetable salad	135
1 glass milk	165
Total	450

Dinner or Supper

	calories
1 medium portion meat or fish	200
1 medium baked potato	100
½ cup string beans	45
1 cup spinach	40
1 slice whole wheat bread (½″ thick)	100
1 apple	100
1 glass milk	165
Total	750

Total for Day:

Breakfast	300
Lunch	450
Dinner (supper)	750
	1500 calories

CHAPTER IX

MATHEMATICS OF BUDGETS

THERE IS AN OLD SAYING to the effect that "if you watch the pennies, the dollars will take care of themselves." Not to be taken too literally, the proverb nevertheless suggests the essential importance of keeping track of expenditures. To spend one's entire income as fast as it is received (often exceeding it, and borrowing until next pay day) is undesirable. This can be avoided if we keep a moderately accurate record of income and expense, together with an approximate and flexible budget pattern; and cultivate a sense of thrift and the habit of saving.

The following discussion gives you suggestions about planned spending, which is the essential purpose of budgeting.

HOUSEHOLD ACCOUNTS

Keeping Records

Records of income and of household and personal expenditures may be kept quite simply, or they may be made very elaborate—the choice will depend upon the individual. Keeping records requires thought and accuracy, but extensive bookkeeping or accounting methods are neither necessary nor desirable. The

purpose is not to keep a set of balanced books, but rather to use a practical device which will assure living within one's income, at the same time deriving the greatest satisfaction from that income. The time required should not be prohibitive, say ten or fifteen minutes a day, and two or three hours at the end of the month.

Record of Income

An accurate record of income, actual or estimated, can be conveniently made as follows:

RECORD OF INCOME								
Item	*Jan.*		*Feb.*		*Nov.*		*Dec.*	
Cash on hand at beginning of year								
Salary (or business income)								
Interest on savings account								
Dividends and interest on investments								
Gifts received								
Miscellaneous income								

Classification of Expenditures

The innumerable personal and household expenditures entailed in managing a family may be classified and grouped in several ways, e.g.:

1. Food:

   Food purchased
   Meals eaten out

2. Shelter:

   Rent
   Property taxes
   Interest on mortgage
   Fire insurance on house
   Repairs and improvements
   Special assessments

3. Household and Operating Expenses:

- Fuel
- Water
- Gas
- Electricity
- Telephone
- Ice
- Laundry
- Household supplies
- Maid service
- Carfare and commutation
- Stationery and postage
- Fire insurance on household furnishings
- Interest on borrowed money
- Maintenance and upkeep (garden, snow removal, etc.)

4. Furnishings and Equipment:

- Furniture
- Rugs
- Draperies
- Bedding
- Linen
- Tableware
- Kitchen utensils and household appliances
- Pictures
- Ornaments
- Flowers
- Sewing machine
- Refrigerator
- Washing machine
- Vacuum cleaner

5. Clothing:

- Ready-made garments
- Materials and trimmings
- Accessories
- Dressmaker
- Cleaning and pressing
- Alterations and mending

6. Health:

- Doctor
- Dentist
- Hospital
- Nurse
- Medicine
- Eyeglasses

7. Education:

- School and college costs
- Books and magazines

8. Recreation:

- Movies
- Theaters
- Lectures
- Concerts
- Radio
- Records
- Clubs
- Travel
- Vacation
- Toys
- Sports
- Hobbies

9. Personal:

- Toilet articles
- Barber
- Hairdresser
- Candy; tobacco
- Perfume
- Jewelry

10. AUTOMOBILE:

- Gasoline and oil
- Repairs
- Garage
- Insurance
- License and taxes
- Depreciation

11. GIFTS:

- Personal gifts
- Church
- Charity
- Public welfare

12. INSURANCE AND TAXES:

- Life insurance
- Accident insurance
- Hospitalization
- Federal income tax
- State income tax

13. SAVINGS:

- Savings account
- Christmas club
- Defense bonds
- Postal savings
- Payments on house
- Investments in stocks and bonds
- Building and loan society
- Cash emergency fund

Form of Record

The simplest possible sort of a record can be kept in an ordinary blankbook by making daily entries, in the appropriate column, of all important items of money received and paid out, noting the nature of the item, as shown:

DATE	ITEM	INCOME		EXPENSES	
3/2	Salary	42	50		
3/4	Rent			57	00
3/5	Telephone			3	20
3/8	Grocer			4	62
3/9	Salary	42	50		
3/12	Shoes			6	95

FOOD

MONTH ____________

DATE	FOOD PURCHASED	AMOUNT	
	Estimate for year		
	Spent to date		
	Estimate for month		

Such a record, however, is not altogether useful unless later transferred to another sheet where the items are classified and totaled. Another form, somewhat more extensive, but not requiring a transfer of the items, is given on pages 183 and 186. Inexpensive booklets like this may be purchased, or an ordinary blankbook may be ruled in the same way. The headings of the columns may be modified to suit individual needs and preferences.

A somewhat different type of record, preferred by some people, is the single-page-for-income-and-for-each-class-of-expenditure type, as suggested on pages 184–5. It will be observed that this plan, providing for "estimates" for the month as well as actual disbursements for each major classification, is more

Daily Expenses

DATE	SALARY AND OTHER INCOME		FOOD		SHELTER		HOUSEHOLD & OPERATING		FURNISHINGS & EQUIPMENT		CLOTHES		HEALTH	
1														
2														
3														
4														
5														
6														
7														
8														
9														
10														
11														
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25														
26														
27														
28														
29														
30														
31														
TOTAL														

Total Income for Month $________　　*Total Expenses for*

Month of ____________________ 194___

DATE	ADVANCE-MENT		RECREATION		PERSONAL		AUTOMOBILE		GIFTS CHURCH CHARITY		INSURANCE & TAXES		SAVINGS	
1														
2														
3														
4														
5														
6														
7														
8														
9														
10														
11														
12														
13														
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21														
22														
23														
24														
25														
26														
27														
28														
29														
30														
31														
TOTAL														

Month $__________ Total Saving for Month $__________

OPERATING EXPENSES			
MONTH ___			
DATE	ITEM	AMOUNT	
	Estimate for year	264	—
	Spent to date	39	75
	Estimate for month	22	—
3/2	Electricity	4	12
3/5	Telephone	5	64
3/6	Laundry	2	19
3/10	Stationery	1	50

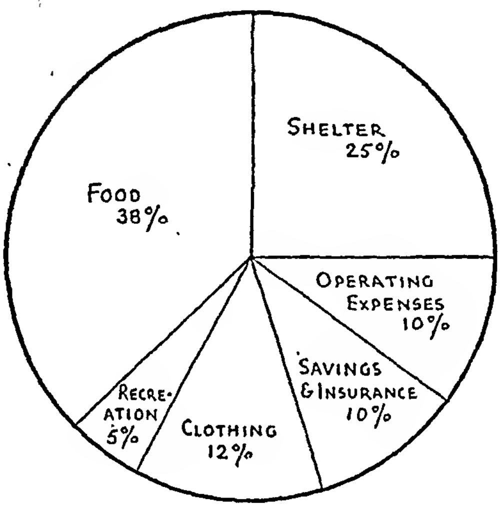
sensitive to actual conditions and probably accomplishes the aim of a budget more successfully than the other schemes. When this type is used, a loose-leaf notebook is usually best, using one page for each classification.

PLANNED SPENDING

Typical Budgets

As already suggested, the essence of budgeting is planning *ahead;* merely keeping a record of actual transactions does not constitute budgeting. Hence, regardless of minor details, the important consideration is the apportioning of expected expenditures in relation to expected income. In short, the "budget ratios" must be chosen with great care. To be sure, conditions

change, and unexpected contingencies will arise, so that a certain degree of flexibility must be maintained. Moreover, individual desires vary considerably, and the suggested "typical" budgets given here are intended only as a practical working guide. They are based on the experience of others, and should prove helpful.



MODEL BUDGET FOR FAMILY OF FOUR
WITH INCOME OF $40 PER WEEK

It is expected that when adopting a scheme of budget ratios, suitable minor variations and adjustments will be made. This is largely a personal matter. However, basic ratios such as about 25% for shelter, and 10% for savings and insurance, should be deviated from as little as possible. It is clear that the two outstanding factors determining these ratios are (1) the total amount of income, and (2) the size of the family. Obviously the amount of income is subject to change from time to time. So, for that matter, is the size of the family, although that occurs less frequently. In such cases of change, it is clear that appropriate adjustments must be made; nor is it advisable to wait until the end of the year to make them.

TYPICAL BUDGET FOR A SINGLE PERSON

Monthly Income	*Shelter*	*Food*	*Clothing*	*Operating Expenses*	*Recreation and Education*	*Savings and Insurance*
\$100	25%	30%	11%	12%	12%	10%
\$150	25%	26%	12%	14%	13%	10%
\$200	25%	23%	12%	16%	14%	10%
\$250	25%	18%	12%	18%	15%	12%
\$300	22%	16%	11%	20%	16%	15%
\$400	18%	12%	10%	24%	18%	18%

TYPICAL BUDGETS FOR A FAMILY OF FOUR

Monthly Income	*Shelter*	*Food*	*Clothing*	*Operating Expenses*	*Recreation and Education*	*Savings and Insurance*
\$100	30%	40%	12%	5%	4%	9%
\$150	25%	38%	13%	8%	6%	10%
\$200	25%	35%	14%	9%	6%	11%
\$250	25%	30%	15%	11%	7%	12%
\$300	25%	27%	15%	12%	9%	12%
\$400	23%	22%	15%	14%	11%	15%

EXAMPLES

1. A family of four has an income of \$48 a week. Approximately how should their income be budgeted?

   4⅓ wks. = 1 mo.

   \$48 × 4⅓ = \$208

   Using the budget ratios for \$200 a mo.:

Shelter	25% × \$208 =	\$ 52.00
Food	35% × \$208 =	72.80
Clothing	14% × \$208 =	29.12
Operating Expenses	9% × \$208 =	18.72
Recreation & Education	6% × \$208 =	12.48
Savings and Insurance	11% × \$208 =	22.88
		\$208.00

2. (a) What maximum monthly rental should be paid by an average family with a yearly income of \$3800? (b) a weekly income of \$37.50?

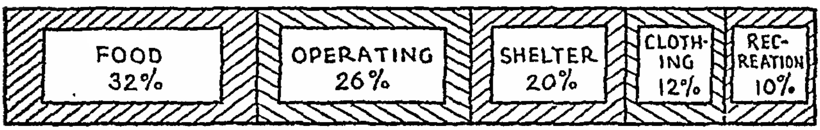
   (a) \$3800 per yr.=about \$316 per mo.;
   at 25% for shelter, $\$316 \times \frac{1}{4} = \$79$ monthly rental, *Ans.*

   (b) \$37.50 per week=\$162.50 per mo.
   $25\% \times \$162.50 = \$40.63$.

   Maximum monthly rental=\$40, or about one week's salary, *Ans.*

3. For a family of four with a monthly income of \$250, approximately how much should the weekly food bill amount to?

   $\$250 \times 30\% = \$75$ per month for food
   $\$\ 75 \div 4\frac{1}{3} = \$17.30$, *Ans.*



ACTUAL AVERAGE EXPENDITURES
OF A GROUP OF FAMILIES
WITH \$2400 A YEAR INCOMES
(How does this compare with the model budget?)

Relation of Budget Ratios to Variation in Income

According to certain economic principles known as Engel's laws, the apportionment of expenditures tends to shift with increased income, as follows:

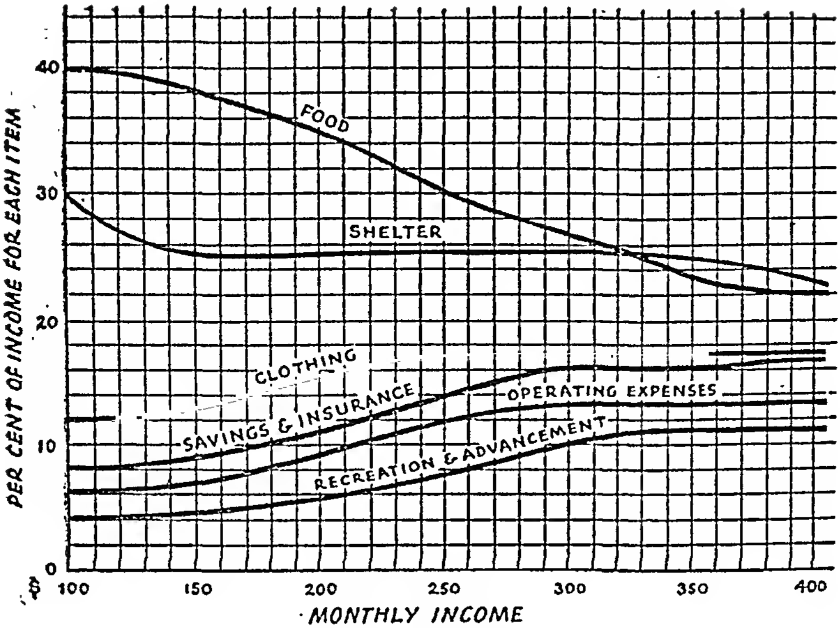
(1) The percentage of total income allotted to *food* decreases.

(2) The percentage allowed for *clothing* remains approximately constant.

(3) The percentage allowed for *shelter* remains approximately constant until relatively high incomes are reached.

(4) The percentage expended for *recreation, luxuries,* etc., increases steadily until the higher incomes are reached, when it "levels off."

(5) The percentages for both *general expenses* as well as for *savings and insurance* continue, in general, to increase.



The Food Budget

For several reasons, the allotment for food should be carefully distributed. Ordinarily, a family spends, or should spend, between one-fourth and one-third of its income for food. If this amount is to be spent most effectively, one should allow:

(a) one pound of fruit and vegetables per day per person.

(b) from ½ lb. to 1 lb. of fats per week per person.

(c) 25% of the total food allowance for bread, cereals, spaghetti, rice, macaroni, etc.

(d) at least as much or more for meat as for milk.

EXAMPLES

1. A family of four with a weekly income of $45 allows 28% of its income for food. (a) How much do they spend for food per person per week? (b) If the wife buys 25 lb. of fruits and vegetables per week at an average cost of 10¢ a lb., what per cent of her food allowance goes for fruit and vegetables?

(a) $\$45 \times .28 \times 1/4 = \$3.15$ per person per wk. for food

(b) $25 \times 10¢ = \$2.50$, weekly for fruits and vegetables
$\$45 \times .28 = \$12.60$, weekly for all food
$\$2.50 \div \$12.60 = .198$, or approximately 20% for fruits and vegetables

2. Mrs. Adams buys an average of 5 qts. of milk per week for her family, at 16¢ a qt. If she allows herself three times as much for meat as for milk, how much does she spend per week for meat? What per cent of her $15 weekly food allowance does she spend for meat?

$5 \times 16¢ = \$.80$
$\$.80 \times 3 = \$2.40$, weekly for meat
$\$2.40 \div \$15 = 16\%$, per cent of total food allowance spent on meat

A typical "breakdown" of an allowance of $50 a month for food might be as follows:

Milk	$ 6
Butter	6
Fruit and vegetables	12
Bread	2
Meat and fish	15
Meals eaten out	9
TOTAL	$50

Eating Out

By way of illustrating the relative expense of eating out with home cooking, consider the following dinner menu for four.

Creole Soup

—oOo—

Roast Shoulder of Lamb

Carrots and Peas   Pan Roast Potatoes

Cole Slaw

—oOo—

Peach Cream Pie

Lemonade

The materials for this dinner would run approximately as follows:

2 cans soup @ 10¢	$ .20
6 lb. lamb shoulder @ 27¢	1.62
2 lb. potatoes @ 4¢	.08
1 bunch fresh carrots	.08
1 can peas	.13
2 lb. cabbage @ 4¢	.08
1 gill cream	.12
3 lb. peaches @ 5¢	.15
Pie mix	.20
Lemons	.10
TOTAL	$2.76

A comparable dinner eaten out might well come to $1.25 per person, or $5.50 for four, including tips (and perhaps a tax). The home cooked meal frequently yields "left-overs," which can taste even better the next day, besides effecting a further saving. It is common practice among commercial restaurants to compute the "unit cost" of preparation of a complete meal (table d'hôte) per person, and then multiply by 2.5 to determine the price to be charged for the dinner. Accordingly, a "dollar dinner" should not cost them more than 40¢ for materials. To be sure, by buying in large quantities they can get a better price than the housewife. But if they can prepare a $1 dinner for 40¢, the housewife can prepare the same dinner for 50¢ or 55¢, and so effect an overall saving of about 50%.

CHAPTER X

MATHEMATICS OF INCOME

In modern society, people derive their income from many different sources. Any money received other than an outright gift or inheritance is considered as income. The larger part of most people's incomes is obtained as wages or salary in return for their labors in the shop, the factory, the store, or the office. Farmers receive their income from the sale of their crops and dairy products. Professional people, like lawyers and doctors, receive fees for their services. Income is sometimes received for the rental of a house or a garage, or as royalty on a book or an invention. Businessmen derive income from the net profit of their business, and salesmen and agents derive theirs frequently in the form of commissions instead of, or in addition to, their salaries.

Income may also be derived from investments. That is, a person invests some of his money in a savings bank, in postal savings, in government bonds, or in stocks and bonds of business corporations. The interest received on such investments constitutes part of their income. Income received from rents, royalties, or interest and dividends on investments, is referred to as "unearned income" for tax purposes; in this connection, "earned income" means wages, salaries, professional fees, etc.

WAGES AND SALARY

Wage Rates

The amount of money received as wages is usually expressed as a rate: thus a laborer may get 30¢ an hour; a mechanic, $4.75 per day; or a typist, $14 per week. When persons are paid by the month, their remuneration is usually referred to as a salary, although we also commonly speak of a "weekly salary of $27.50." A clerk in an office may earn a salary of $125 a month, and a business executive's salary may amount to $4800 per year; in the first case, the clerk is usually paid twice a month, and in the latter, the executive receives his money once a month.

EXAMPLES

1. An employee in a factory is paid at the rate of 54¢ an hour. How much does he earn in a week, if he works 44 hours per week?

   $\$.54 \times 44 = \$23.76$, *Ans.*

2. If a mechanic's assistant is hired at $5.25 per day, how much will he earn in a year, allowing 300 working days? How much is this a month?

   $\$5.25 \times 300 = \$1575$, *Ans.*

   $\$1575 \div 12 = \$131.25$, *Ans.*

3. A nurse in a doctor's office receives a salary of $22.50 weekly. How much does she earn on a monthly basis?

   We allow $4\frac{1}{3}$ weeks to the month,
   since $12 \times 4\frac{1}{3} = 52$, the number of weeks in a year.
   $\$22.50 \times 4\frac{1}{3} = \$97.50$, *Ans.*

Piecework

In various kinds of business, notably manufacturing, workers are sometimes paid according to how much they produce rather than on the length of time they work. This is called a *piecework* basis.

EXAMPLE

A craftsman working in a toy shop is paid 22½¢ for each toy he makes. His record for the week was:

Mon.,	24 toys	Thurs.,	27 toys
Tues.,	26 "	Fri.,	25 "
Wed.,	23 "	Sat.,	11 "

How much did he earn that week?

$24+26+23+27+25+11=136$, total no. of pieces.

$\$.225 \times 136 = \$30.60$, *Ans.*

Earnings of Factory Workers

Wage rates in industry vary in different states, and with economic conditions. Average weekly earnings in the factories of New York State are given in this table:

Year	*Weekly Wages*	*Year*	*Weekly Wages*
1925	\$28.26	1933	\$21.83
26	29.02	34	23.19
27	29.30	35	24.36
28	29.44	36	25.34
29	29.99	37	27.36
30	28.81	38	26.29
31	26.42	39	27.29
32	22.73	40	28.40

Annual Incomes

The distribution of family incomes in the United States is as shown:

Under \$1000	44%	of all families
\$1000—\$2000	35%	" " "
\$2000—\$3000	12%	" " "
\$3000—\$4000	4%	" " "
\$4000—\$5000	2%	" " "
over \$5000	3%	" " "
	100%	

The National Income

In a recent year, the total national income received, approximately $72,000,000,000, was derived from various sources as follows:

Employees' compensation	68.2%
Profit from private enterprise	15.8%
Interest from investments	13.0%
Rents and royalties	3.0%
	100.0%

Further analysis revealed that in that year approximately 75% of the country's total income was labor income, and that 25% represented all the income derived from productive business enterprise and ownership of property.

BONUS AND COMMISSION

Overtime Work

In many industries and communities the maximum number of hours or days of labor are specified by law or otherwise. When an employee works in excess of these specified hours, he is said to work overtime. There are many occasions requiring overtime work: holiday seasons in stores; end of the month, or quarterly periods in business offices; storms, accidents, or other castastrophes for repairmen, etc. The wage-rate paid for overtime work is often one-and-a-half times the regular rate. Thus if a worker puts in 2 hours extra after a 7-hour working day, and his regular rate is 60¢ an hour, his wages for the day may be computed in either of two ways:

(a) 7 regular hours + 2 overtime hours =
$7+(2\times 1\frac{1}{2})=10$ regular hrs.
$60¢\times 10=\$6$

(b) Regular day's pay = \$4.20
2 hr. overtime at 90¢ = 1.80
Total = \$6.00

Piecework Quotas

When employed on a piecework basis, a worker may be expected to produce a minimum number of pieces per day. This number is called his quota. If he produces more than his quota, he may be paid a bonus of so much more per piece than the regular rate.

Straight Commission

In many lines of business, salesmen work on a "straight commission basis." This means that remuneration is based upon the number of items sold in a given period of time, or upon the total amount of sales. The commission is usually expressed as a certain percentage of the amount of sales, although sometimes it is given as so much per item.

EXAMPLES

1. A book agent receives $.38 for each book he sells. If he sells an average of 50 books a week, what are his average weekly earnings?

   $.38×50=$19 per week, *Ans.*

2. A paint salesman receives a commission of 7½% on all his sales. If his monthly volume of sales averages $3200, what is his average monthly commission?

   $3200×7½=$240 per month, *Ans.*

Commission on a Graduated Basis

The commission paid to a salesman is sometimes based on a "sliding scale" of rates, varying with the quality or price-range of the merchandise, or upon the amount sold.

EXAMPLES

1. Mr. Dickson received a commission of $4 on each of the first 20 vacuum cleaners sold in a month, $4.50 on each of the next 10 sold, and $5 on each one above 30. If in a given month Mr. Dickson sold 34 vacuum cleaners, how much did he earn that month?

$$\begin{aligned} \$4\ \ \ &\times 20 = \$80 \\ \$4.50 &\times 10 = \ \ 45 \\ \$5\ \ \ &\times\ \ 4 = \ \ \underline{20} \\ & \qquad \$145, \textit{ Ans.} \end{aligned}$$

2. An agent is commissioned to sell hair brushes on the following scale of commissions:

Brushes listed at $ 8 a doz.	......	10% commission
" " " $10 "	......	12% "
" " " $15 "	......	15% "
" " " $18 "	......	20% "

During a given period the agent's orders were as follows: 120 doz. @ $8; 95 doz. @ $10; 50 doz. @ $15; 30 doz.@ $18. What was his total commission?

$$\begin{aligned} 120\times\$\ 8\times10\% &= \$\ 96.00 \\ 95\times\$10\times12\% &= \ 114.00 \\ 50\times\$15\times15\% &= \ 112.50 \\ 30\times\$18\times20\% &= \ \underline{108.00} \\ & \quad \$430.50, \textit{ Ans.} \end{aligned}$$

Salary and Commission

Salesmen and store clerks are frequently paid on the basis of a combined salary and commission.

EXAMPLES

1. Miss Riley, a salesgirl in a department store, receives a salary of $22.50 a week plus a commission of 3% on all sales for the week over $450. How much will she earn in a week when her total sales amounted to $748?

$$\begin{aligned} &\$748-\$450=\$298 \\ &\$298\times\ .03=\$8.94 \\ &\$22.50+\$8.94=\$31.44, \textit{ Ans.} \end{aligned}$$

2. A traveling salesman receives a salary of $45 a week and a commission of 2½% on all sales. If his total sales last year amounted to $74,600, what was his total income that year?

$$\begin{aligned} \$74{,}600\times2\tfrac{1}{2}\% &= \$1865, \text{ commission} \\ \$45\times52 \qquad &= \ \underline{2340}, \text{ salary} \\ & \quad \$4205, \text{ total income, } \textit{Ans.} \end{aligned}$$

3. Mr. Hilton is a salesman whose salary is \$200 a month, and who also receives a commission of 6% on all sales over \$2500 a month. In October, his sales amounted to \$2942; in November, \$2375; and in December, \$3486. (a) What did Mr. Hilton's total commission amount to in these three months? (b) What was his average monthly income during this period?

(a) Oct.: 6%×\$442=\$26.52 commission
Nov.: none
Dec.: 6%×\$986= 59.16 commission
\$85.68, total commissions, *Ans.*

(b) \$200×3 =\$600.00, salary
Total commissions= 85.68
3)\$685.68
\$228.56, average monthly income, *Ans.*

Agents and Brokers

Many things are sold through the efforts of a third party, acting between buyer and seller. Thus real estate is usually bought and sold through an agent, or a licensed real estate broker. Similarly, most insurance is sold by an insurance agent or a broker; stocks and bonds are sold by brokers. Farm products and other commodities are frequently sold through a *commission merchant,* or middleman, who acts as agent for the purchaser as well as the seller. The middleman usually receives the produce, stores it temporarily if necessary, makes the sale, delivers the goods, pays all incidental expenses, deducts his commission, and remits the balance to the farmer or producer whose product was sold. His commission is based upon the gross receipts before the expenses are deducted.

EXAMPLE

A commission merchant sold 500 bbl. of apples for his client at \$4.25 per bbl. He charged a commission of 4½% and paid the trucking charges of \$85.40 and storage charges amounting to \$32.50. Insurance was ½% of the value of the shipment. What net proceeds

did the commission merchant remit to his client? What was his commission?

$$500 \times \$4.25 = \$2125, \text{ gross receipts}$$
$$\$2125 \times 4\tfrac{1}{2}\% = \$95.63, \text{ commission, } Ans.$$

Expenses:

Trucking	$ 85.40
Storage	32.50
Insurance (½%×$2125)	10.63
Commission	95.63
	$224.16

$$\$2125 - \$224.16 = \$1900.84, \text{ net proceeds, } Ans.$$

BANK CHECKING ACCOUNTS

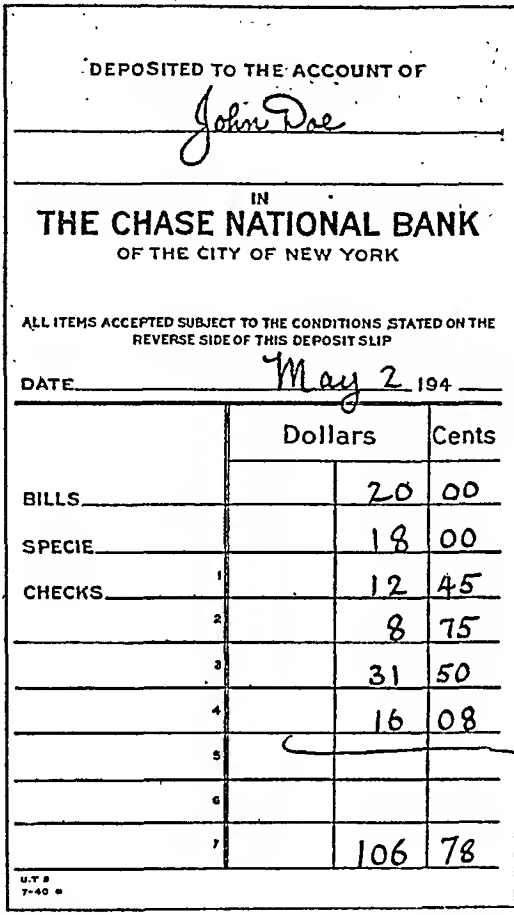
Bank Deposits

Money received as gross or net income is generally deposited in a bank. Money so deposited may be put into a savings account or special interest account, where it draws interest at from 2½% to 3½%. The manner of computing interest on such accounts is fully described in Chapter XX. Money may also be deposited in a Christmas savings fund, where it generally draws no interest and cannot be withdrawn until toward the end of the year.

Checking accounts pay no interest, but money may be withdrawn at any time by means of checks. The use of checks in business is an invaluable convenience; about 90% of all business transactions are handled by checks or other credit instruments instead of actual cash. Many people prefer to use checks in paying their private and household bills, taxes, insurance premiums, and rent.

Depositing Money

Whenever money is deposited, whether in a *savings* or *checking account,* the money is turned over to the *teller* and is accompanied by a *passbook* and a *deposit slip*. The passbook



DEPOSITED TO THE ACCOUNT OF

John Doe

IN

THE CHASE NATIONAL BANK

OF THE CITY OF NEW YORK

ALL ITEMS ACCEPTED SUBJECT TO THE CONDITIONS STATED ON THE REVERSE SIDE OF THIS DEPOSIT SLIP

DATE May 2 194

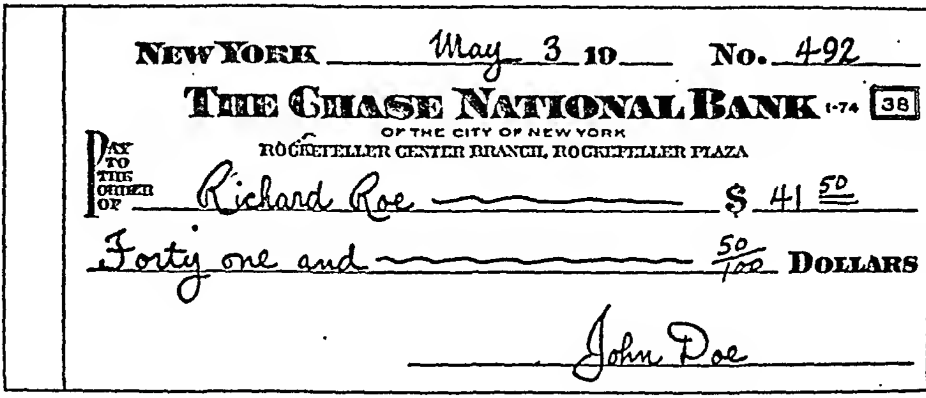
		Dollars	Cents
BILLS		20	00
SPECIE		18	00
CHECKS	1	12	45
	2	8	75
	3	31	50
	4	16	08
	5		
	6		
	7	106	78

U.T.
7-40

belongs to the depositor, and is his permanent, cumulative record of all deposits in his checking account, and of both deposits and withdrawals in his savings account. The teller files the deposit slip, and makes the appropriate entry in the passbook.

Writing Checks

Not only is the use of a check a convenient way of transferring money—it is also a safe way. A typical check, properly filled out, is shown herewith.



NEW YORK May 3 19 No. 492

THE CHASE NATIONAL BANK 1-74 38

OF THE CITY OF NEW YORK

ROCKEFELLER CENTER BRANCH, ROCKEFELLER PLAZA

PAY TO THE ORDER OF Richard Roe $ 41 50

Forty one and 50/100 DOLLARS

John Doe

Important Points to Be Remembered

When making out a check, the following points should be kept in mind:

1. Never write a check in pencil—always use ink.
2. Never erase on a check—write another one.
3. Never date a check on a Sunday or a legal holiday—date it a day in advance.
4. Make your figures distinct, and your writing legible.
5. Write the amount in figures *close* to the dollar sign.
6. Write the amount in words by beginning at the *extreme left* of the check, and fill in the space not used with a wavy line.
7. Don't write a check for less than a dollar.
8. Don't sign *blank* checks.

Endorsing a Check

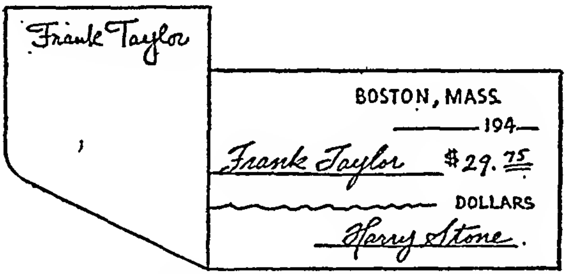
The person (or firm) to whom a check has been drawn is known as the *payee.* When the payee signs his name on the back of the check, he is said to have *endorsed* the check. Such an endorsement, i.e., merely signing one's name on the back as it appears on the face of the check, constitutes an "endorsement in blank," and virtually makes the check as good as cash to anyone else into whose hands it may fall. Hence,

unless a check is about to be deposited, various forms of endorsement are used.

(1) *Endorsement in Blank*

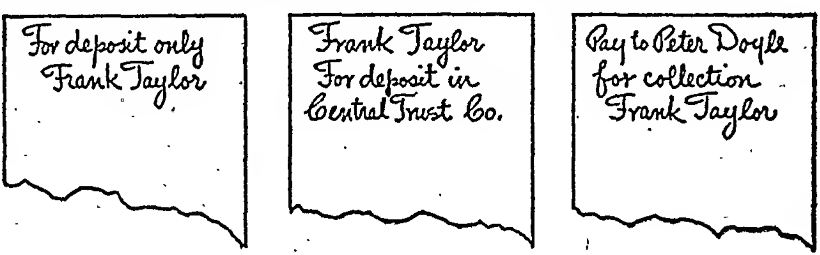
(2) *Endorsements in Full*

An *endorsement in full* indicates the name of the person, firm, or organization to whom the check is being transferred; such person, by endorsing it in full again, may transfer the ownership to a third party, and so on.

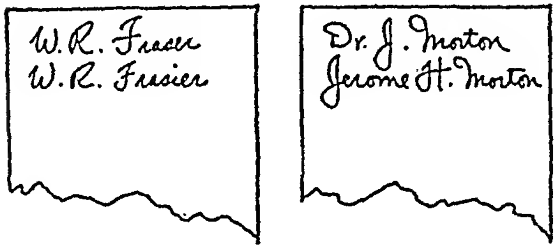


(3) *Restrictive Endorsements*

A *restrictive endorsement* deliberately limits the negotiability of the check, indicating the special purpose for which it is to be used, or the particular party for whom it is intended.



(4) *Double Endorsements*



Occasionally, because of an error in the way the payee's name has been written on the face of the check, or in order to conform with the payee's signature at the bank, a *double endorsement* is used. The check is first endorsed as written on the face, and then, beneath, as it should have appeared on the face.

Cost of Maintaining a Checking Account

Most commercial banks require a minimum average balance of from \$200 to \$500 in their regular checking accounts. The depositor loses the interest on such average minimum balance as may be required, and in addition pays a nominal monthly charge for the checking account. This fee usually depends upon the number of deposits made and the number of checks drawn, as well as the actual average balance maintained during the month. For ordinary personal checking accounts this charge may amount to about \$2 per month or slightly more. In recent years many banks have inaugurated a "special checking department," in which no minimum balance is required, and no charge is made for deposits. A book of 20 checks is furnished at 10¢ a check, and when these are used up, additional checks may be bought at 10¢ each. A variation of this scheme is to charge 5¢ for each check drawn and 5¢ for each item deposited.

CHAPTER XI

MATHEMATICS OF HOME OWNERSHIP

OWNING ONE'S OWN HOME has many advantages. It is an excellent form of security. There are, however, certain responsibilities connected with the ownership of property. In this chapter we shall deal with some of the financial problems involved, and in the following chapter with some of the practical problems of managing a house and garden.

From the standpoint of economics, owning a house calls for sound financial management. There is far more to it than simply the initial cost of a house and lot. Two of the largest items, as we shall soon see, are the mortgage and the property taxes. In addition there are numerous operating expenses, generally referred to as "carrying charges"; these include insurance, special assessments, water taxes, fuel, maintenance and repairs, replacements, depreciation, and interest on the investment.

When compared with the cost of renting a house or an apartment, the total cost of owning and running a home may be greater than, equal to, or less than that of renting. Each individual case must be studied on its own merits, and in the light of specific conditions. In deciding on whether to buy a house, not only the economic factors, but many other non-financial and personal matters must also be considered.

BUYING A HOUSE

Real Estate

Land alone may be purchased, with the intention of building a house on it; or a house already built, new or old, may be purchased. In both cases the property is referred to as real estate. When a house is bought or sold, the land on which it stands is nearly always included with the house. When buying undeveloped land (i.e., a plot without a house), the price may be quoted as "$2000 for a lot 25'×100'," or as "$.60 per square foot," or as "$72 a foot frontage."

Real estate is usually bought and sold through licensed agents called *brokers*. When property is transferred from one person to another, the broker receives a commission for his services, generally 5% of the sale price. This commission is paid by the seller of the property, not by the purchaser.

EXAMPLES

1. Mr. Greene bought two adjoining lots, each 25'×100', at $65 a front foot. How much did he pay for them?

   The "frontage" refers to the width, since this is usually the dimension that fronts on the street or avenue.

   $ 65×25=$1625, price per lot.
   $1625× 2=$3250, total cost, *Ans.*

2. A broker sold Mr. Ketchum's house and lot for $9600, receiving a commission of 5%. How much did Mr. Ketchum receive for his property?

   $9600× 5%=$ 480, broker's commission.
   $9600—$480=$9120, amount received, *Ans.*

Purchasing Expenses

Buying real estate is by no means as simple as buying other forms of property, such as a piano, a refrigerator, or an automobile. The property may have to be surveyed. The title to the property must be searched, and the mortgage recorded. Thus legal fees, bankers' fees, and incidental items inevitably arise.

2. Mr. Tompkins bought a house for \$12,400, paying \$5000 cash, with a mortgage for the balance at 5%, payable semiannually, together with reduction of principal on each interest date in the amount of \$300 until the mortgage was reduced to \$4400. (a) How much must he pay during the first 1½ years? (b) How long will it be until the mortgage is reduced to \$4400? (c) How much interest must he pay annually thereafter?

(a) \$12,400—\$5000=\$7,400, amount of mortgage

$\$ 7400 \times \frac{5}{100} \times \frac{1}{2} = \$185$, interest, 1st 6 mo.
300, principal reduction
\$485, total payment, 1st interest date

$\$7100 \times \frac{5}{100} \times \frac{1}{2} = \$177.50$, int., 2nd 6 mo.
300.00, principal reduction
\$477.50, total payment, 2nd interest date

$\$6800 \times \frac{5}{100} \times \frac{1}{2} = \$170$
300
\$470, total payment, 3rd interest date

Total for 1½ yr.=\$1432.50

(b) \$7400—\$4400=\$3000.
\$3000÷\$ 600=5 years, time required to reduce principal to \$4400, *Ans.*

(c) $\$4400 \times \frac{5}{100} = \$220$, annual interest after 5 years, *Ans.*

Amortization of Principal

Any systematic reduction in the amount of the principal of a debt is called a process of amortization. A debt may merely be reduced by amortization, as in the second illustrative example above, or the amortization may be continued until the debt has been completely wiped out. While this procedure has been commonly used for years in business and financial matters, it has only recently become popular in connection with mortgages or real estate. The vogue, moreover, is rapidly increasing. A typical illustration follows:

i.e., the nominal owner, who transfers his property conditionally as security for the loan. In the example given, the "owner" of the house has an *equity* of only ¼, i.e., $1500÷$6000. When the amount "on mortgage" is finally paid off, the mortgage is void, and the purchaser owns the property free and clear—he then has an equity of 100% in the property. If, however, he cannot pay the mortgage loan when due, the mortgagee (i.e., the lender) has a right to "foreclose"; this is a legal proceeding by which the mortgagor's right of redeeming the property is lost.

The traditional mortgage was usually granted by a bank, a builder, or a private investor, for an amount usually not in excess of 60% to 70% of the value of the property. The loan generally ran from 3 to 5 years, with interest at 5½% or 6%, payable semiannually or quarterly. At the end of that time it was generally renewed for another term of 3 years. Sometimes it was not renewed, but left "open", i.e., the mortgagee continued the loan from year to year, but reserved the right to demand his principal on any interest date after due notice. Not infrequently the mortgagee requested (or the mortgagor volunteered) reduction of the principal according to some arrangement mutually agreed upon.

EXAMPLES

1. Mr. Adams bought a house and lot for $7650, paying $3150 in cash, and giving a mortgage for the balance at 6%, payable quarterly. (a) What per cent of the purchase price was the mortgage? (b) What amount of interest must he pay every 3 months? (c) How much interest will he pay in 10 years on this mortgage?

(a) $7650 purchase price
   3150 cash
$4500, amount of mortgage

$\frac{\$4500}{\$7650} = 58.8\%$, *Ans.*

(b) $\$4500 \times \frac{6}{100} \times \frac{1}{4} = \$67.50$, quarterly interest, *Ans.*

(c) $\$4500 \times \frac{6}{100} \times 10 = \$2700$, ten years' interest, *Ans.*

PROPERTY TAXES

The Tax Rate

Real estate, whether developed or not, is subject to taxes by state and local governments. A committee of the legislature prepares a budget of estimated expenditures for the ensuing year. When this budget has been approved and enacted into law, the tax commission determines the necessary tax rate to be levied in order to raise the required amount. A condensed typical budget of a fairly large city is presented herewith:

ANNUAL ESTIMATE, 194–

EXPENDITURES	
Public Works	$ 1,519,000
Department of Safety and Health	1,651,000
Board of Education	2,400,000
Administration	1,822,000
Maintenance	584,000
Total	$ 7,976,000
DEBT SERVICE	4,721,000
STATE AND COUNTY TAX	1,592,000
	$14,289,000
CREDITS AND REVENUES	1,896,000
	$12,393,000

All the real estate in the community is valued by a special bureau known as the tax assessors. This value, which is revised (up or down) from year to year, is called the assessed valuation of the property. It is usually somewhat less than the market value, although the reverse is sometimes true. If a property owner feels that his property has been assessed unfairly he has the right to ask for a review, and frequently an adjustment is made.

The entire amount to be raised by the community is then divided by the total assessed valuation of *all* the property in

EXAMPLE

(A) Old Method (Fixed Mortgage)	(B) New Method (Amortization Plan)
1. Home-owner borrows $4000.	1. Home-owner borrows $4000.
2. He pays $220 interest annually at 5½%.	2. He pays $40 quarterly, in addition to interest at 5½%, to reduce the principal.
3. Total interest paid during succeeding 25 years=$5500.	3. Total interest paid during next 25 years=$2777.50.
4. At the end of 25 years the home-owner still owes $4000.	4. At the end of 25 years the home-owner owes nothing.

To be sure, during the first 18 years of the 25-year period, the home-owner must make larger annual payments under plan B than under plan A; but during the last 7 years or so the total payments each year are actually less under plan B than under plan A. Under the amortization plan, during the first year the total payments amount to:

$216.70,	interest
160.00,	principal
$376.70,	total.

During the second year, they amount to $367.90; each succeeding year they decrease by $8.80, so that during the 25th year the total is only $165.50.

But it must be remembered that the debtor has saved $4000 over a period of 25 years,—a sort of compulsory saving.

Amortized mortgages are clearly very desirable from everyone's point of view. For the home-owner, they mean "automatic" thrift; for the bank and the private investor, they mean minimum impairment of investment principal. A somewhat more practical and more commonly used amortization plan, illustrated toward the end of the chapter, provides for *constant* monthly or quarterly payments instead of decreasing payments.

lowered, and the amount of his tax might be the same as it was the year before; and, of course, it might be larger or smaller.

The annual amount of the tax is usually due and payable in quarterly installments; if payments on any quarter (except the first) are made in advance, a discount of 2% per annum is sometimes allowed. If taxes are "delinquent", i.e., not paid until past due, a fine often running as high as 7% per annum is imposed. If property taxes are allowed to accumulate unpaid, for a considerable number of years, the community government can sell the property for the amount of overdue taxes plus the interest thereon.

CARRYING CHARGES

Mortgage and Taxes

We have seen something of the expenses involved in the purchase of a house. Let us now consider the "carrying charges", or expenses involved in owning and maintaining a house from year to year.

The two outstanding items, so far as the amounts are concerned, are (1) the interest on the mortgage, together with principal payments, if amortized; and (2) the property taxes. Consider an average one-family house in the suburbs, worth perhaps $8000, and carrying a fixed mortgage of $4800 at 5%. The annual mortgage interest would amount to $240; no amortization is provided. The taxes on such a house in an average community might run in the neighborhood of $270 per year, depending, of course, upon the location.

Maintenance and Repairs

There are other expenses to be included, however. Water consumption is often taxed separately; fuel (coal, gas, or oil) for heating is required; replacements and improvements must be made; repairs, painting, etc., must be taken care of.

the community; this determines the tax rate for that year. For example, in the city whose budget was given above, the total assessed valuation of all real property amounted to \$294,000,000, in round numbers; \$12,393,000÷\$294,000,000= .04215, or \$4.22 per \$100 of assessed valuation. Obviously, the sum of all property taxes will equal the total amount to be raised for the budget. Occasionally, the amount charged per \$100 or per \$1000 slightly exceeds the actual rate, which results in collecting a little more money than is actually required; this excess is called a tax *overrun.*

Finding the Tax on Property

The tax rate may be expressed in several ways:

(a) a rate of \$3.82,—which means \$3.82 per \$100 of valuation.

(b) a rate of 38.2¢,—which means 38.2¢ per \$1000 of valuation.

(c) a rate of 3.82¢,—which means 3.82¢ on the dollar.

(d) a rate of 38.2 mills,—which means 38.2¢ per \$1000 (since 1 mill=$\frac{1}{10}$¢)

EXAMPLES

1. Mr. Coster's house and lot are assessed at \$7850. If the tax rate is \$2.94, what is the amount of his tax?

$$\$7850 \div \$100 = 78.5$$
$$78.5 \times \$2.94 = \$230.79, \textit{ Ans.}$$

2. A piece of real estate has an assessed valuation of \$22,000. If the tax rate is 21.3144 mills, find the amount of the tax.

$$21.3144 \text{ mills} = \$.0213144$$
$$\$.0213144 \times \$22{,}000 = \$468.92, \textit{ Ans.}$$

It will be seen that the amount of tax to be paid by a property owner varies from year to year, and can be changed in one of two ways, or both: by changing the tax rate, and by changing the assessed valuation of his property. Thus the tax *rate* might increase slightly, but the valuation might be

Suppose a man buys a home for \$7000, paying \$2000 in cash, and giving a mortgage of \$5000 for the balance. He plans to pay off this mortgage loan over a period of 17 years, in accordance with the above amortization schedule. The table shows how his monthly carrying charge would be calculated.

Mortgage (\$7.03×5)	\$35.15
Taxes	22.00
Fuel	8.00
Maintenance	10.00
Interest on investment (@ 3%)	5.00
Depreciation (2% of \$7000 annually)	11.67
TOTAL MONTHLY CARRYING CHARGES	\$91.82

Other Expenses

These include insurance, depreciation, special assessments, and interest on the cash invested in the purchase price. To continue our "average suburban house", the annual costs might run as follows:

Purchase price:	$8000
Cash:	$3200
Mortgage:	$4800

Annual Operating Costs:

Interest on mortgage	$ 240
Taxes	270
Special assessment	80
Insurance	12
Oil	95
Water	20
Repairs	135
Depreciation	160
Interest on cash investment (@3%)	96
Total	$1108

MONTHLY CARRYING CHARGE..........$1108÷12=$92.33

Carrying Charges, Including Amortization

As already suggested, it is coming to be common practice to amortize mortgage loans. The following table gives the *fixed* monthly combined payments of interest and principal required to amortize a loan of $1000 at 4½% interest in various periods of time:

Monthly Payment	*Time Required*
$ 6.33	20 years
7.03	17 "
7.65	15 "
8.48	13 "
10.37	10 "
13.91	7 "

UPKEEP OF A HOUSE

Repairs, Replacements, and Improvements

As anyone who owns a house will agree, the need of repairs and replacements is inevitable. Materials will wear out; and things will become broken or used up. The exterior of a wooden house must be painted at regular intervals, every few years; roofs leak, and must be repaired or recovered; floors must be refinished; rooms redecorated; screens must be replaced after a few years (unless made of copper, when their "useful life" is much longer); outdoor wooden steps must be replaced eventually. Not only is there the ever present problem of repairs and replacements, but the desirability of occasional improvements as well. A new fence here; a handy cabinet to be built there; some extra shelving needed in the attic or the cellar; and so on.

How Much Lumber?

A little careful figuring beforehand will often save time and money when buying lumber for a repair job or a home-built project.

EXAMPLES

1. The front porch of Mr. Morley's house is $11\frac{1}{2}$ ft. wide and 28 ft. long. He wishes to recover the floor completely. How many pieces of flooring will be needed, if it comes in 15-ft. and 18-ft. lengths, $3\frac{1}{2}$ in. wide?

   If 15-ft. lengths were used, there would be too much waste. Using 18-ft. lengths, both full lengths and half-lengths can be utilized.

   Suppose the porch were 18 ft. wide. Since it is 28 ft. long, and the pieces are $3\frac{1}{2}$ in. wide, it would require

   $\frac{28\times12}{3\frac{1}{2}}$, or $28\times12\times\frac{2}{7}=96$ lengths of flooring, each 18 ft.

   But since it is a little less than 12 ft. wide, and half-lengths can be used, the number of pieces needed equals $\frac{2}{3}\times96$, or 64 lengths, *Ans.*

CHAPTER XII

MATHEMATICS OF HOUSE AND GARDEN

AMONG THE MANY SATISFACTIONS derived from the ownership of a home is the opportunity of decorating it to suit yourself, of beautifying the grounds, and of maintaining the house in sound and presentable condition. To do these things requires a miscellany of knowledge about materials and supplies, where they may be purchased, and how much will be required for the job to be done. Here is where a bit of mathematics usually comes up. To be sure, most of the applications of mathematics to problems of "tinkering" around the house—repairing a cabinet, designing book shelves, or building a chicken coop—involve primarily the use of simple measuring instruments and ordinary mechanics' tools. This, in turn, demands not only skill in simple computation, but also familiarity with the common systems of measurement and weight. It may be necessary, for example, to convert a quantity from one unit to another, as yards to inches, or pints to gallons, etc.

In the present chapter we shall suggest a few typical problems that arise in connection with a house and garden, and which illustrate the kinds of computation that are frequently demanded of the home-owner.

Mixture	*Proportion by parts cement: sand: stone*	*Amounts required for 1 cu. yd. of Rammed Concrete*		
		**Cement (bbl.)*	*Sand (cu. yd.)*	*Stone (cu. yd.)*
Rich	1 : 1½ : 3	1.91	0.42	0.85
Standard	1 : 2 : 4	1.51	0.45	0.89
Medium	1 : 2½ : 5	1.24	0.46	0.92
Lean	1 : 3 : 6	1.06	0.47	0.94

*One bbl. of cement = 4 cu. ft.

Mr. Forbes decides to use the medium mixture. Since he needs 15 cu. yds. of concrete, he will have to use:

Cement ....$15 \times 1.24 = 18.6$, or 19 bbl.
Sand ....$15 \times 0.46 = 6.9$, or 7 cu. yd.
Stone (gravel) ....$15 \times 0.92 = 13.8$, or 14 cu. yd.

Painting and Roofing

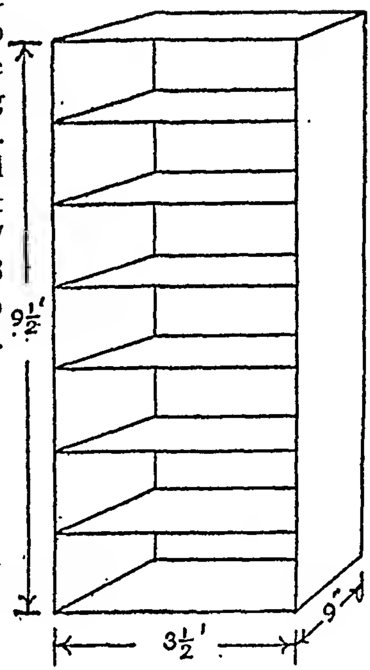
The following examples illustrate the types of computation involved in keeping the exterior of a house or other building in proper condition to protect it against weather conditions.

EXAMPLES

1. A flat-roofed storage shed, 58 ft. long, 42 ft. wide, and 28 ft. high, is to be given two coats of paint. It is estimated that for the first coat, one gallon of paint will cover 400 sq. ft. of surface, and for the second coat, one gallon will cover 600 sq. ft. Allowing 400 sq. ft. for doors and windows, find the cost of painting at \$3.50 a gallon.

Perimeter $= 2\,(58+42) = 200$ ft.
Total area (4 walls) $= 200 \times 28 = 5600$ sq. ft.
Roof area $= 58 \times 42 = 2436$ " "
8036 sq. ft.
Allowance for openings $= 400$ " "
7636 sq. ft.

2. A man wants to build a simple storage cabinet of shelves, with no backing or trim, and having the dimensions shown. The 9″ shelving comes in 10′, 12′, and 15′ lengths. How many feet of lumber should he buy? How much will it cost at 8½¢ a foot? Since there are 7 shelves and a top-piece, he needs 8 pieces each 3½ ft. long. He also needs 2 side-pieces each 9½ ft. long.



2 10-ft. lengths (for the sides) = 20 ft.
2 15-ft. lengths (for the shelves) = 30 ft.
50 ft., total

50×8½¢=$4.25, *Ans.*

Cement and Concrete

Occasionally a cement walk, or runway, or a concrete wall or flooring is to be laid. The following problem illustrates some of the necessary calculations.

EXAMPLE

Mr. Forbes wishes to lay a cement and gravel driveway to his garage. The driveway is to be 9 ft. wide and 90 ft. long, and is to be laid to a depth of 6 in. How many cubic yards of concrete must he prepare? How much gravel, sand, and cement are required?

$$\frac{9\times 90\times \frac{1}{2}}{27}=15 \text{ cu. yd. of concrete.}$$

To determine the amount of cement, sand, and gravel, we refer to the following table of various mixtures:

Painting and Papering

EXAMPLES

1. A painter asks \$25 to "decorate" the walls of a living room 14′ ×22′ and a bedroom 12′×18′. The ceilings are 9½ ft. high. The owner decides to paint the walls himself, using "cold water" paint, giving the living room 2 coats of Nile green, and the bedroom 1 coat of dusty pink. A gallon of cold water paint will cover about 550 sq. ft., and requires a special brush costing \$2.00. At \$2.10 a gallon, how much will it cost him to do the job himself? How much will he save by hiring a mechanic at \$4.50 to do the job for him?

Total surface of bedroom walls (no allowance for doors and windows)=2(12+18)×9½=570 sq. ft.
Hence 1 gal. pink paint is required.

Total surface of living room walls (no allowance for openings)= 2(14+22)×9½=684 sq. ft.

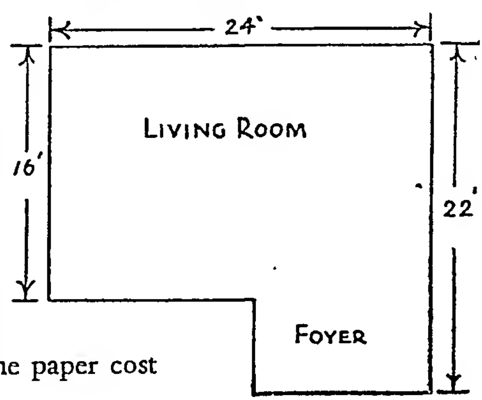
684×2=1368 sq. ft. (for 2 coats)

1368÷550=2.5 gal., or 3 gal. of green paint (½ gal. not sold).

Total Cost: Pink paint	@	\$2.10=	\$2.10
Green "	@	2.10=	6.30
Brush		=	2.00
			\$10.40, *Ans.*

\$10.40+4.50=\$14.95
\$25—\$14.95=\$10.05, saving.

2. A living room and foyer have the following dimensions, with a 10½-ft. ceiling. Allowing 100 sq. ft. for doors and windows, how much wallpaper is required for the walls of these two rooms? What will the paper cost at \$2.25 a roll?



$7636 \div 400 = 19.1$ gal. (first coat)
$7636 \div 600 = \underline{12.7}$ gal. (second coat)
31.8 gal., total; practically, 32 gal. required

$\$3.50 \times 32 = \$112$, *Ans.*

2. Roofing is generally sold by the *square,* which is equivalent to an area of 100 sq. ft. What is the cost of covering a gable, or A-shaped roof, with roofing material at \$6.85 a square, if each side of the roof is 42 ft.$\times 18\frac{1}{2}$ ft.?

$42 \times 18\frac{1}{2} \times 2 = 1554$ sq. ft., or approx. $15\frac{1}{2}$ squares.

Usually a fractional part of a square is not sold. Hence 16 squares are required.

$16 \times \$6.85 = \$109.60$, *Ans.*

3. Wooden roof shingles are usually sold in bundles of 250 each. Allowing 2.5 bundles per square, find the cost of shingling an A-shaped roof, each side of which is 52 ft.$\times$21 ft., at \$14.50 per thousand? (Fractions of a bundle are not sold.)

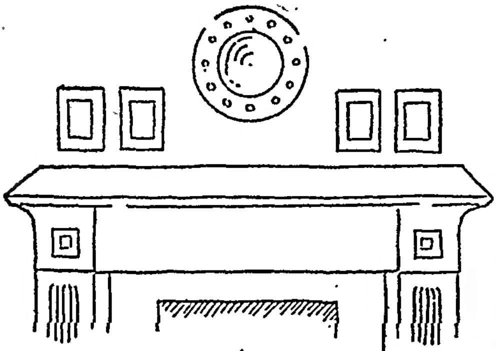
$52 \times 21 \times 2 = 2184$ sq. ft., or 21.8 squares.
$21.8 \times 2\frac{1}{2} = 54.5$, or 55 bundles.

$$\frac{55 \times 250}{1000} \times \$14.50 = \$199.38, \textit{ Ans.}$$

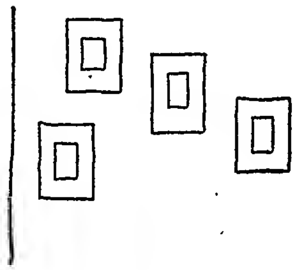
INTERIOR DECORATION

In considering the interior of a home, several types of activities are involved. Some have to do with "basic" decoration, i.e., painting and papering walls, the ceiling, and the treatment of floors. Others have to do with the selection and arrangement of furniture, while still others have to do with the choice and use of accessories, such as shades, blinds, drapes, rugs, lamps, pictures, and decorative ornaments. Some of these involve no mathematics at all. Others make use of simple arithmetic, and less frequently, of a few geometric principles, such as symmetry, parallelism, etc.

Similarly, in arranging furniture, or when hanging pictures, symmetric arrangements are usually most pleasing, although *asymmetry,* or off-balance, is also pleasing at times. Thus four equal-sized, similarly-framed pictures might be hung in several ways, according to individual taste:



A



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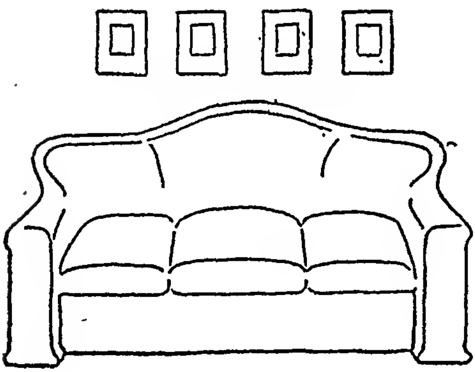

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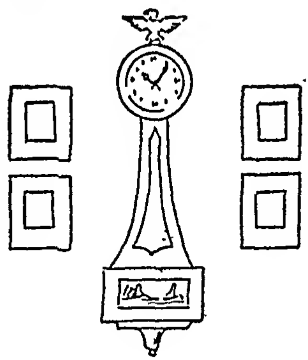


D

THE GARDEN AND GROUNDS

Whether the grounds around a house include gardens, shrubbery, etc., or not, the use of grass in contributing to the beauty of the surroundings is practically universal. A lawn may be planted by sowing seed, or it can be made by laying sod.

Wallpaper is usually sold by the roll, each roll 18 in. wide and 16 yd. long. Hence 3 rolls are generally allowed for every 200 sq. ft. of surface to be papered.

Combined Perimeter$=16+24+22+24+6=92$ ft.

$92\times10\frac{1}{2}=966$ sq. ft., including openings.

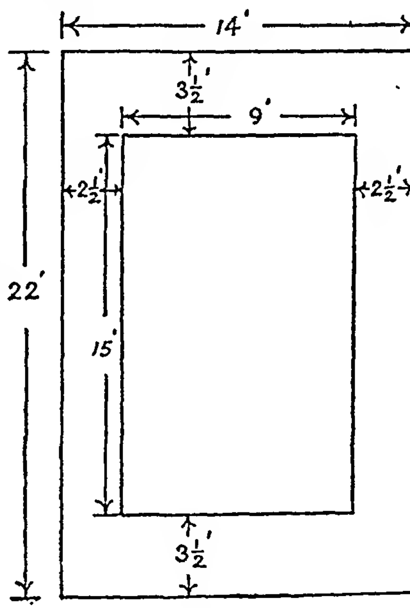
$966-100=866$ sq. ft., excluding openings.

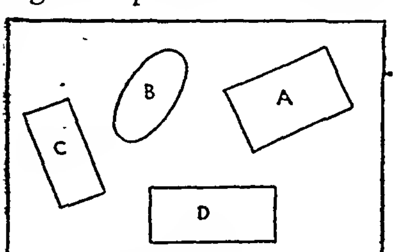
$866\div200=4.3$ rolls, or 5 rolls; no fractions of a roll.

$\$2.25\times5=\$11.25$, *Ans.*

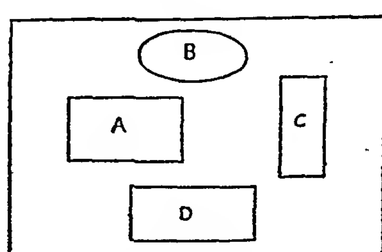
Use of Geometry

To some extent, an appreciation of geometric form, of symmetry and balance, is helpful in connection with the arrangement of furniture and accessories. For example, a rectangular rug, 9′×15′, is to be laid on a living room floor 14′×22′. How should it be placed to look best? Unless there is some special reason to the contrary, leaving an equal "border" all around is generally most acceptable, as shown herewith. Small rugs look best when laid so that the edges are *parallel* to the walls.





WRONG WAY



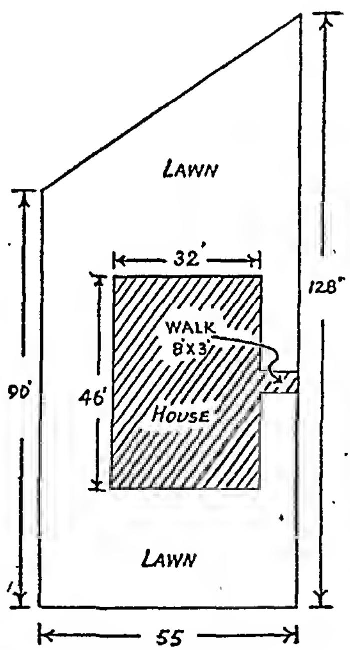
RIGHT WAY

10 lb.	seed	@	$5.00
5 lb.	"	@	2.60
3 lb.	"	@	1.60
1 lb.	"	@	.55
			$9.75 (instead of 2 ten-lb. bags for $10)

100 lb.	enricher	@	$5.00
50 lb.	"	@	3.25
25 lb.	"	@	2.00
10 lb.	"	@	1.00
			$11.25; or, better still, two 100-lb. bags for $10.

Total Cost: $9.75+$10=$19.75, *Ans.*

When buying sod, the pieces are generally squares 9″×9″, at so much a hundred. How much sod would be required to lay a lawn as shown in the accompanying figure, allowing for a gravel walk 8 ft. long and 3 ft. wide?



Area of entire plot (trapezoid)=
$\frac{1}{2}(55)(90+128)=5995$ sq. ft.

Area of house=
$46\times 32 \quad =1472$ sq. ft.

Area of walk=
$8\times 3 \quad = \quad 24$ sq. ft.

Area of lawn $\quad =4499$ sq. ft.,
or 4500 sq. ft.

Each "square" of sod 9″×9″, or $\frac{3}{4}' \times \frac{3}{4}'$, which gives an area of $\frac{9}{16}$ sq. ft.

Hence, $4500 \div \frac{9}{16} = 8000$ squares of sod, or 80 C, *Ans.*

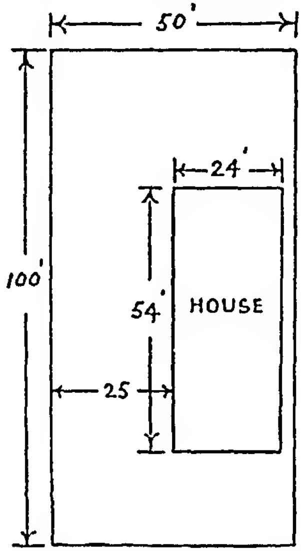
At $2.50 a C, the sod would cost $2.50×80, or $200, *Ans.*

Flower Beds, Walks, etc.

In laying out the ground, planning the garden, or landscap-g, problems such as the following are frequently encountered.

EXAMPLES

1. Mr. Easton's house, 54 ft. by 24 ft., is situated on a "double lot," 50 ft.$\times$100 ft., as shown in the diagram. How many sq. ft. of



lawn area must he plant, disregarding the narrow strip between the house and the property line?

$100\times50=5000$ sq. ft.
$25\times54=1350$ " "
3650 sq. ft., *Ans.*

2. A garden catalog quotes lawn seed at the following prices:

1 lb.	3 lbs.	5 lbs.	10 lbs.	25 lbs.	100 lbs.
55¢	$1.60	$2.60	$5.00	$12.00	$45.00

It recommends that 1 lb. should be used for every 200 sq. ft., and also suggests the use of lawn "enricher," 10 lbs. to be applied to every 200 sq. ft., at the following prices:

5 lbs.	10 lbs.	25 lbs.	50 lbs.	100 lbs.
55¢	$1.00	$2.00	$3.25	$5.00

How much will it cost Mr. Easton to plant a lawn?

$3650\div200=18\frac{1}{4}$, no. of lbs. of lawn seed required.
$(3650\div200)\times10=182\frac{1}{2}$, no. of lbs. of "enricher" required.

CHAPTER XIII

MATHEMATICS OF THE FAMILY CAR

NOT ONLY DOES the United States lead the world in the production of automobiles, but the per capita number of cars is greater than in any other country. The family car is a characteristic feature of the American scene.

Many matters connected with the automobile involve mathematics. Its design and construction, which are problems of engineering; its operation, problems of science and technology; its costs, problems of economics and finance. In this chapter we shall deal exclusively with the last of these, especially from the viewpoint of the car-owner. We shall touch also on some questions of safe driving.

We usually think of the total expense of a car as consisting of two parts: (1) the initial cost of the car, and (2) the cost of operating the car. Under the first heading we include the initial purchase price, the trade-in allowance, if any, and the interest on the investment. The operating expenses include the license fee, the garage rental, the gasoline and oil consumed, tire replacements, repairs and servicing, insurance, depreciation, and interest on the investment.

EXAMPLES

1. A driveway 66 ft. long and $5\frac{1}{2}$ ft. wide is to be paved by laying gravel to a depth of 4 in. What will the gravel cost at \$3.50 a load?

   Volume of gravel needed: $66 \times {}^{11}\!/_{2} \times {}^{1}\!/_{3} = 121$ cu. ft.
   $121 \div 27 = 4.5$ cu. yd., or $4\frac{1}{2}$ loads.
   $4\frac{1}{2} \times \$3.50 = \$15.75$, *Ans.*

2. A gardener wishes to lay out a circular flower bed 28 ft. in diameter. If the "top soil" is to be filled in to an average depth of 6 in., how much will he need, and what will it cost at \$2.75 a load?

   $$\text{Area} = \frac{\pi D^2}{4} = {}^{22}\!/_{7} \times \frac{(28)(28)}{4} = 616 \text{ sq. ft.}$$

   At a depth of 6 in., volume $= 616 \times \frac{1}{2} = 308$ cu. ft.
   $308 \div 27 = 11.4$, or $11\frac{1}{2}$ loads.
   $11\frac{1}{2} \times \$2.75 = \$31.63$, *Ans.*

Other Problems in Gardening

Both the amateur and professional gardener occasionally have to use simple mathematics in connection with such problems as constructing cold frames; laying out patterns for flower beds; computing proportions in formulas for mixing various grades of fertilizers, using percentages of nitrogen, phosphorus or other soil deficiencies, preparing solutions for sprays and insecticides, and similar problems.

Buying a Car on Time

More than half of all new cars sold are bought on an installment purchase plan (see Chapter XIX). For example, if Mr. Jordan were to buy the above car on time, the transaction might be something like this:

Price, f.o.b., Flint, Mich.	$ 975.00
Freight charges	27.85
Accessories	48.90
Total	$1051.75
Less trade-in allowance	190.00
Total cash price (without tax)	$ 861.75
Federal tax	34.13
	$ 895.88

Down payment	$370.00
Installments (12×$45)	540.00
Total installment price	$910.00
Federal tax	34.13
	$944.13

The difference between $910 and $861.75, or $48.25, represents the additional amount that must be paid for the privilege of buying on time; it is a so-called "carrying charge", and represents the cost of consumer credit. If Mr. Jordan buys the car for cash, he must part with $895.88; if he buys it on the installment plan, he must pay $404.13 at the time of purchase ($370 down payment plus $34.13 tax), and $45 a month for 12 months, or $540 more in deferred payments, a total of $944.13.

OPERATING EXPENSES

License Tax

The automobile license must be paid each year, and is therefore properly regarded as an operating expense. However, it is a direct tax upon the car before it can be legally operated.

COST OF OWNING A CAR

Initial Purchase Price

An inviting advertisement announces a new model for \$975. But the thoughtful purchaser must realize that the cost of owning a car involves more than the initial purchase price. To begin with, the price quoted, \$975, is usually "f.o.b.", meaning freight on board; this means that the freight charges from the factory to the purchaser's residence must be paid by the purchaser, and must be added to the \$975. Then there is the Federal tax on the purchase of automobiles, which at the present time amounts to 3½%. And there are usually a few "accessories" which are purchased at the time the car is bought.

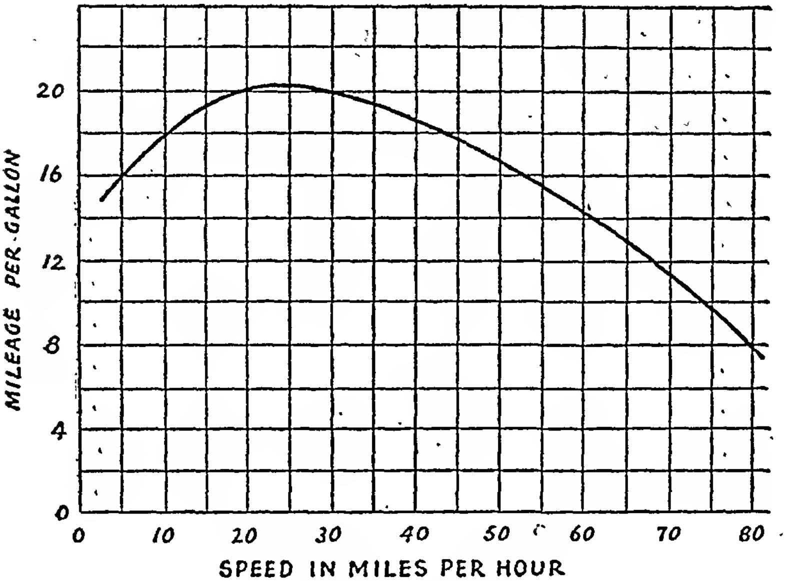
Trade-In Allowance

If the purchaser of a new car already owns a used car, the dealer usually allows a certain amount on the old car as a "trade-in value". This allowance varies with the make of car and with the age of the car; the older the car, the less the allowance. The decreased value on account of age is not, however, directly proportional to the age. Thus a car worth \$1000 when new might be worth, for purposes of trade-in, about \$600 a year later; when 2 years old, \$450; when 3 years old, \$400; when 4 years old, \$350; when 5 years old, \$200; and thereafter the allowance might not be more than \$150.

EXAMPLE

Mr. Jordan's purchase of a car entailed the following items:

Price, f.o.b. Flint, Mich.	\$ 975.00
Federal tax, at 3½%	34.13
Freight charges	27.85
Accessories	48.90
	\$1085.88
Trade-in allowance	190.00
Net cost (cash basis)	\$ 895.88



EXAMPLES

1. A man takes a trip of 870 miles. If his car averages 15 miles to the gallon, (a) how much will it cost for gas and oil, if gasoline costs 17¢ per gallon, and oil 32¢ a quart? (b) What is the cost of fuel per mile?

   (a) $870 \div 15 = 58$ gal. gasoline
   $58 \times 17¢ = \$9.86$, cost of gasoline.
   $870 \div 100 = 8.7$, or 9 qt. oil
   $9 \times 32¢ = \$2.88$, cost of oil.
   $\$9.86 + \$2.88 = \$12.74$, total cost, *Ans.*

   (b) $\$12.74 \div 870 = \$.015$, or about $1\frac{1}{2}¢$ per mile, *Ans.*

2. Mr. Jordan drives his car an average of 12,000 miles per year. What is the total annual cost of gas and oil, if he gets 16 miles to the gallon, and pays on the average 18¢ a gallon for gasoline and 35¢ a quart for oil?

   $12{,}000 \div 16 = 750$; $750 \times 18¢ = \$135$, cost of gasoline
   $12{,}000 \div 100 = 120$; $120 \times 35¢ = \$42$, cost of oil
   $\$135 + \$42 = \$177$, total annual cost of fuel, *Ans.*

This tax, levied by each state, is usually based upon the weight of the car. In New York State, for example, the rate is as follows:

50¢ for each 100 lb. or major fraction thereof up to 3500 lb.;

75¢ for each 100 lb. or major fraction thereof above 3500 lb.

EXAMPLE

If Mr. Jordan's car weighed 3750 lb., what is the amount of the license fee?

$$
\begin{array}{l}
50¢ \times 35 = \$17.50 \\
75¢ \times \ 2 = \underline{\ \ 1.50} \\
\qquad\qquad \$19.00, \text{ total license fee.}
\end{array}
$$

NOTE: 50 lb. is not a "major fraction" of 100 lb.

Garage Rental

This is another item of expense, one which might also be regarded as an operating expense, although it must be paid even if a car is not in use. If a car is not to be used during the winter, this expense can be reduced by putting the car in "dead storage". Average garage rental runs from $5 to $10 per month. If Mr. Jordan paid $6.50 per month for his garage, his yearly rental would cost 12×$6.50, or $78.

Gas and Oil

The cost of gasoline and oil consumed in operating a car is one of the more obvious operating expenses, although it is not as large a proportionate part of the total operating cost as is sometimes thought; this will become clearer as we go on.

Depending upon the design, weight, number of cylinders, etc., a car may consume roughly from 5 to 10 gallons of gasoline in going 100 miles; or, as it is usually expressed, the "mileage" is from 10 to 20 miles on a gallon. An "average car" will operate at 16 or 17 miles per gallon. Similarly, the oil consumption may be estimated at about 1 quart for every 100 miles of driving.

EXAMPLE

A car originally costing \$1150 was turned in at the end of 5 years for a trade-in allowance of \$200. Find the annual depreciation.

\$1150—\$200=\$950, total depreciation.

\$950÷5=\$190, annual depreciation, *Ans.*

Interest on the Investment

This is an item of expense frequently overlooked. If Mr. Jordan purchased his car on a cash basis (\$895.88), he might presumably withdraw \$896 from his savings account with which to pay for it. If he did, he would naturally lose the interest on that sum, which, at 3%, would amount to \$26.88 per year; this should be added to the yearly cost of owning and operating it.

Cost Analysis

In summarizing, let us analyze the expense entailed by Mr. Jordan in owning and operating his own car. Assume that he purchased the car for cash (\$896), and drove it for 5 years, at the end of which time he turned it in for a new car, receiving an allowance of \$205 for the old one. We shall assume also that during these 5 years he drove his car a total distance of 57,600 miles, as shown by the speedometer. How much did it cost him per year to own and operate his car?

Initial cost	\$896.00
Operating Costs for 5-Year Period	
License fee (5×\$19)	\$ 95.00
Garage rental (@ \$6.50 per mo.)	390.00
Gasoline (16 mi./gal., @ 18¢)	648.00
Oil (576 qt. @ 35¢)	201.60
Tires (11 tires @ \$22.50)	247.50
Repairs and servicing	246.75
Insurance (\$70 per yr., average)	350.00
Depreciation (\$896—\$205)	691.00
Interest on investment (at 3%)	134.40
Total operating cost	\$3004.25

\$3004.25÷5=\$600.85, annual cost.

Other Operating Costs

In addition to gas and oil, and garage rent and license fees, there are the occasional repairs that are inevitable as a car grows older, or the repairs that are due to minor mechanical accidents; the washing and servicing; the replacement of batteries and bulbs, of new tires or tubes, etc. These cannot be readily estimated beforehand, although they can be approximated, and a careful record should be kept of all such items. Tires, for example, if of a reasonably good quality, may be guaranteed for 10,000 miles, and may actually give 12,000 miles of service or more. By "rotating" 5 tires, this "set" is good for about $\frac{5}{4} \times 12{,}000$, or 15,000 miles. If a car is driven 10,000 miles a year, a set of tires must be replaced every year and a half.

Insurance

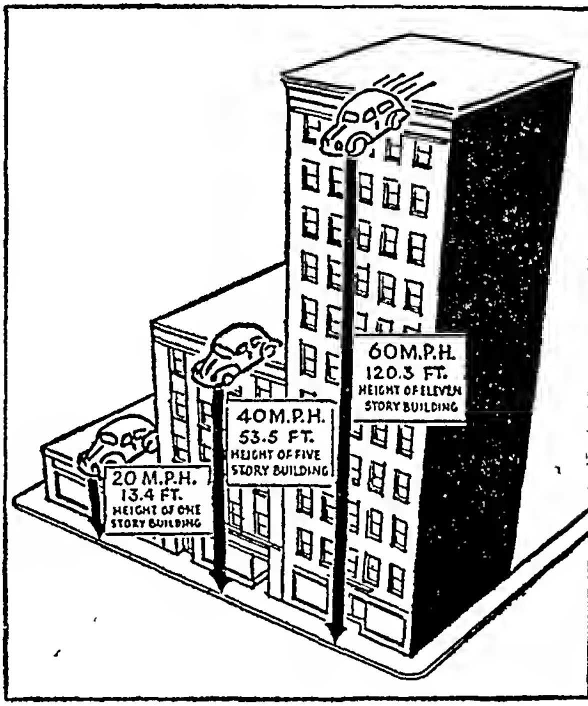
Automobiles are insured primarily against public liability, property damage, fire and theft, and collision. The details of automobile insurance are discussed in Chapter XXIII. The rates vary with the car, the protection guaranteed, and the location of the owner's residence. Typical coverage might cost about $75 a year in premium payments.

Depreciation

As already suggested, the value of a car decreases as it grows older, no matter how good the condition in which it is kept. In fact, a car that is only a few days old, or that has been run only a couple of hundred miles, is in reality a used car, and has depreciated considerably in value. The depreciation of a car does not, strictly speaking, follow a "straight-line" depreciation formula. When a car is one year old, it may be assumed to have depreciated 40% in value; it will not, however, be worthless in $2\frac{1}{2}$ years ($2\frac{1}{2} \times 40\% = 100\%$). For practical purposes the annual depreciation may be figured as follows: subtract the trade-in allowance from the original total cost, and divide by the number of years of service.

per hr. has a hitting force equivalent $\frac{1}{2}(3000)(44)^2$, since 30 miles per hr. equals 44 ft. per sec.; or slightly more than 2,900,000 foot-pounds of energy per second, or 5280 horsepower. Moreover, since the speed is *squared* in this formula, doubling the speed gives *four* times the energy. The significance of this may be better understood from the following table; the figures in the right hand column indicate the equivalent "crashing effect" for a car going at the speed indicated, just as if the car had fallen from the height indicated:

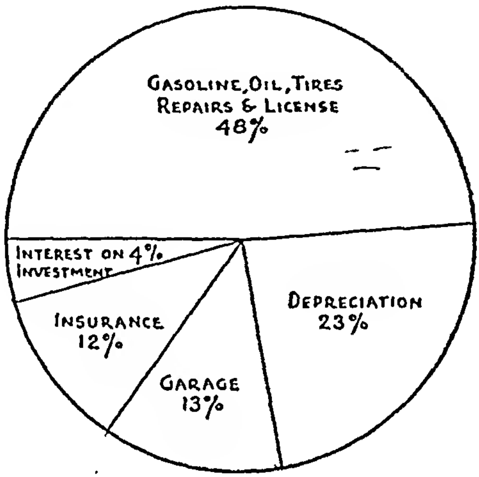
Speed in mi. per hr.	*Fall in feet*
20	13
30	30
40	54
50	83
60	120



Stopping a Car

The "reaction time" of a person is the length of time required for a stimulus to cause a response. Under normal conditions, the average reaction time for muscular control is about $\frac{3}{5}$ of a second; $\frac{2}{5}$ of a second is good. A tired driver's

This cost analysis can also be shown graphically, as in the accompanying circle-chart, where the relation of the separate costs to one another is clearly shown. In short, it can be safely said that it costs, under ordinary conditions, about $50 a month to own and operate a car. Of course, conditions vary; it costs more to operate a car in the city than in the country



PERCENTAGE ANALYSIS OF ANNUAL COST OF OWNING AND OPERATING A CAR

or the suburbs; if one has a garage on his own property, the monthly garage rental is saved.

It might also be noted that a rough rule is as follows: the annual cost of owning and operating a car is about ⅔ of the original cost of the car. This holds good of a more expensively priced car, since nearly all the operating-cost items are also higher for a more expensive car.

SAFETY IN DRIVING

Force of a Heavy Moving Object

In mechanics it is shown that the kinetic energy of a moving body is equal to $\frac{1}{2}mv^2$, where m represents its weight and v its speed. Thus a car weighing 3000 lb. and moving at 30 miles

CHAPTER XIV

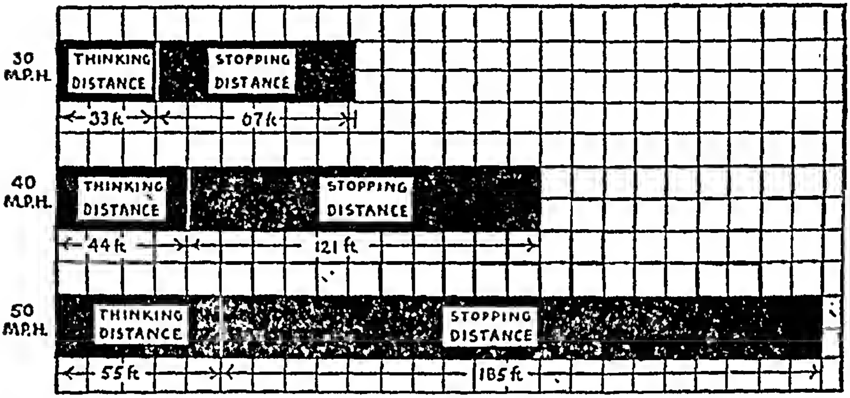
MATHEMATICS OF TRAVEL

IN PRIMITIVE TIMES, man traveled on foot or on horseback. On foot a man can cover about three miles an hour; on horseback, he can travel perhaps five or six times as fast. A hundred years ago, the standard method of transportation was by stagecoach, which averaged no more than eight miles an hour. By 1865, or a little later, steam trains were in general operation in the eastern part of the United States, and traveled at 20 miles an hour. By the turn of the century, the few automobiles then in existence rarely did better than 20 miles an hour, although rail travel had become considerably speedier. Today, ordinary steam railway trains commonly average from 45 to 65 miles an hour, while the modern streamlined, Diesel-powered trains average from 90 to 100 miles per hour. Travel by motor car, whether touring or by bus, commonly averages from 30 to 45 miles an hour, depending upon traffic conditions.

Meanwhile, travel by water, and latterly by air, have both made great strides. From small sailing vessels and clipper ships to steam packets; then huge screw propeller ocean liners; and finally turbo-electric-driven "floating cities" capable of crossing the Atlantic in 3½ days instead of as many months. In the

reaction time may be $\frac{4}{5}$ of a second. Assuming that the average reaction time of a fairly alert driver is $\frac{1}{2}$ second, then at 20 mi. per hr. a car goes 15 ft. before the motorist applies his brakes; similarly:

at 30 mi./hr., a car goes 22 ft.,
" 40 " " " " " 29 ft.,
" 50 " " " " " 37 ft.,
" 60 " " " " " 44 ft.



DISTANCE REQUIRED TO STOP A CAR UNDER ORDINARY CIRCUMSTANCES

To be somewhat more conservative, we shall assume an average reaction time of $\frac{3}{4}$ second; then for various speeds, the distance required to stop a car is vividly shown by the graph.

Round Trip Fares from New York to:	In Coaches	In Pullmans
Brunswick, Ga.	$28.05	$45.35
Charleston, S. C.	22.60	36.20
St. Augustine, Fla.	31.55	50.40
Miami, Fla.	40.35	65.15

The variety of accommodations available in standard Pullman sleeping cars, together with the range in prices, may be seen from the following table of rates:

Where lower berth rate is $2.10 . . .

Upper berth rate is	$1.60
Section for 1 person (one bed) is	2.80
Section for 2 persons (one bed) is	3.25
Roomette for 1 person is	3.15
Roomette for 2 persons is	3.60
Duplex single room for 1 person is	3.40
Duplex single room for 2 persons is	3.80
Bedroom for 1 person is	3.80
Bedroom for 2 persons is	4.20
Bedroom suite for 2 or more persons is	7.35
Compartment for 1 person is	4.20
Compartment for 2 persons is	6.30
Drawing room for 1 person is	5.25
Drawing room for 2 persons is	7.35

If the lower-berth rate is higher than $2.10, the rates for the other accommodations are in proportion. The variety of tickets available is illustrated by the following quotations, based on passage from New York to Los Angeles, San Francisco, Portland, or Seattle and return:

2-months first-class ticket for only	$135.00
2-months coach ticket	90.00
60-days coach-intermediate class ticket	101.25
60-days first-intermediate class ticket	123.05
3-months first-class ticket	139.75
3-months first-intermediate class ticket	123.45
3-months coach-intermediate class ticket	101.85

air, man's triumphs are still fresher. Within a short span of years, the simple flying machine of the Wright Brothers has developed into the powerful but graceful airliner, that can cross the continent at more than 200 miles an hour.

In this chapter we shall discuss travel by rail and bus, as well as by steamship and plane.

TRAVELING BY RAIL

Railway Fares

The cost of traveling by rail is based fundamentally upon the distance to be covered, and is calculated at a certain rate per mile, usually in the neighborhood of from 2 to 2½ or 3¢ per mile. The "fare" is understood to mean the basic cost of the journey; additional conveniences, such as meals, private compartments, and sleeping accommodations are extra.

Occasionally, there is a moderate reduction in the rate if a "round trip" ticket is bought; this is not, however, the general rule in connection with railway fares. Tickets are usually good for 30 days, with stop-over privileges. Children under five, when accompanied by parent or guardian, are entitled to free passage. If over 5 but under 12, they are charged half-fare, and if over 12, full fare. Railroads often run excursion trips, especially for comparatively short runs, during the summer season, winter sports season, etc. Such excursions are usually offered at attractive reductions in rates; for example, the regular fare, one way, from New York to Asbury Park, is $1.20 per person, but on an excursion fare the rate per person for a round trip is $1.25, a reduction of practically 50%.

With regard to sleeping accommodations, there is a considerable range of variation, depending upon the nature of the accommodations. Some notion of the additional cost may be gleaned from the following comparison:

improvement of accommodations other than sleeping arrangements; air-conditioning of cars; and a reduction in costs, including meals. A typical illustration is the following, which relates to the "stream-liner" run from Chicago to San Francisco:

SCHEDULE

Every Third Day: Streamliner "City of San Francisco."

Same Date Each Month										*No. 101*	
2	5	8	11	14	17	20	23	26	29	7:45 P.M.	Lv. Chicago (C.&N.W. Ry.)
3	6	9	12	15	18	21	24	27	30	3:30 A.M.	Lv. Omaha (U.P.R.R.)
3	6	9	12	15	18	21	24	27	30	6:05 P.M.	Lv. Ogden (S.P. Co.)
4	7	10	13	16	19	22	25	28	*	9:30 A.M.	Ar. San Francisco (S.P. Co.)

*Arrival on the 31st of a 31-day month, or on the 1st following a 30-day month.

The fare for this trip, a distance of about 2270 miles, is $39.50, or about $1\frac{3}{4}$¢ per mile.

TRAVELING BY BUS

Advantages

Within the last 10 or 15 years, traveling by bus has become commonplace. At first limited to short runs, bus routes now crisscross the nation. The two chief advantages offered by bus travel are cheaper rates, and flexibility of the routes followed. There are some disadvantages, because bus trips take more time, and on long trips may be somewhat less comfortable.

Costs

We give below typical rates charged by a large bus line; the fares quoted are per person, from New York to the cities stated:

	One-way	*Round-trip*
Albany	$ 1.95	$ 3.55
Boston	3.25	5.85
Buffalo	5.75	10.35

Railroad Time Tables

A typical railway time table is shown below:

Miles		★ Hell Gate Express	★ The Federal	★ The Bunker Hill	★ The Mayflower	★ The Penn-Bay State	★ The Bay State	★ The Pilgrim	★ The Bostonian	★ Yankee Clipper
		178	172	6	8	♦188	10	182	14	22
		Daily	Daily	Ex. Sun.	Daily	Daily	Daily	Daily	Daily	Daily
		AM	AM	AM	AM	AM	AM	AM	AM	N'ON
0.0	New York Grand Central...Lv.	.....	.....	7 00	8 00	.....	9 00	.....	11 00	12 00
.....	New York 125th St. (See Note).			7 10	8 10		9 10		11 10	.....
.....	New York Penna. Sta.	12 45	2 10	.....	.....	8 45	.....	10 00	.....	.....
33.1	Stamford	.....	.....	6c55	8 46	9 33	.....	10 48	11 17	c12 19
41.0	South Norwalk	.....	.....	7c07	8c29	9 43	.....	10c30	11c29	c12 31
56.6	Bridgeport	2 14	.....	8 07	9 11	10 02	10 07	11 15	12 07	c12 55
72.3	New Haven ....Due	2 35	3 40	8 25	9 30	10 20	10 25	11 35	12 25	1 20
73.3	New Haven ....Lv.	2 50	3 50	8 30	9 35	10 30	10 30	11 40	12 30	1 25
105.6	Saybrook ....Due	.....	.....	.....	10 14	.....	.....	12 17	1 06	.....
123.1	New London	3 50	4k50	9 27	10 37	11 27	11 27	12 38	1 28	2 22
141.6	Westerly	4 28	.....	9 53	11 07	11 53	11 53	1 04	1 54	.....
158.3	Kingston, R. I.	4 40	.....	.....	11 27	.....	.....	1 24	2 14	.....
185.3	Providence	5 23	6 05	10 40	11 58	12 40	12 40	1 55	2 45	3 30
.....	Boston Back Bay	6 40	7 05	11 35	12 55	1 35	1 35	2 50	3 40	4 25
229.1	Boston South Sta. ......Due	6 45	7 10	11 40	1 00	1 40	1 40	2 55	3 45	4 30
		AM	AM	AM	PM	PM	PM	PM	PM	PM

EXAMPLES

1. If the one-way railway fare from New York to Boston is $4.60, what is the rate per mile?

   From the above time table, the distance is 229 miles.

   $4.60÷230=$.02, or 2¢ per mile.

2. The "Trail Blazer" leaves New York at 6:25 P.M. and arrives in Chicago at 10:25 A.M., a distance of 908 miles. If the round trip fare is $27.25, find (a) the rate per mile, and (b) the average speed maintained.

   $27.25÷908=$.03

   $.03÷2=1½¢ per mile, *Ans.* (a)

   From 6:25 P.M. to 10:25 A.M.=17 hours
   (time difference=1 hr.)

   908÷17=53.4 mi. per hr., *Ans.* (b)

Modern Streamlined Travel

In recent years railroad travel has been modernized in a number of ways, including the introduction of high-speed, Diesel-powered. articulated trains (some having 17 cars); the

(b) how much longer by motor coach.
(c) the average speed of the train.
(d) the average speed of the bus.

(a) From 10:00 to 8:00=10 hrs.; adding one hour for change from Central time to Mountain time=11 hrs.; less 10 min. stop at Ellis=10 hrs. 50 min.

(b) From 12:15 P.M. to 12:15 A.M.=12 hrs.; from 12:15 A.M. to 3:55 A.M.= 3 hr. 40 min. more; adding 1 hr. for change in time, and deducting 45 min. for stopovers at Topeka, Salina, and Ellis, the total time is 15 hr. 55 min.

15 hr. 55 min.
10 hr. 50 min.

5 hr. 5 min., longer by bus.

$5\frac{1}{12}$ hr.$\div 10\frac{5}{6}$=.47, or 47% longer, or almost half again as long by bus as by rail.

(c) Average speed of train$=\dfrac{640}{10\frac{5}{6}}=59.1$ mi. per hr.

(d) Average speed of bus$=\dfrac{640}{15\frac{11}{12}}=40.2$ mi. per hr.

Standard Time Belts*

A word or two of explanation concerning "change of time". Since the earth rotates from west to east, the sun appears to "rise" in the east and "move" toward the west. The distance from New York to San Francisco, some 3000 miles, or about 50° of longitude, represents a difference of about $3\frac{1}{3}$ hours in time. This means that the sun rises in San Francisco $3\frac{1}{3}$ hours later than it does in New York. To avoid any confusion that this situation might cause persons who were traveling a considerable distance, the railroads in 1883 adopted a system of reckoning time that is now known as "Standard Time." According to this system, the territory of the United States is divided into four time belts. Each belt covers approximately 15° of

*From Schaaf: *Progressive Business Arithmetic.* Courtesy D. C. Heath & Co.

	One-way	*Round-trip*
Chicago	$12.35	$19.80
Pittsburgh	6.60	11.90
Washington	3.30	5.95
Miami	15.90	28.65

The round-trip by bus from New York to Boston, for example, at $5.85, compares with twice $4.60, or $9.20, for the round trip by rail; a saving of something over 36% The same railroad, interestingly enough, offers a special *excursion rate* of $3.50 for the round trip, which must, however, be made there and back on the same day, and is only offered on Sundays during the summer months.

Comparison with Rail Travel

By way of showing the comparison in time required, as well as the change in passing from one time belt to another, we give the following time schedule for the run from Kansas City to Denver, by rail (Union Pacific R.R.) and by motor coach:

Elevation in feet	*Miles*			*Motor Coach*	*Pacific Limited*
748	0	Lv Kansas City	(Central time)	12:15	10:00
880	68	Ar Topeka	(" ")	2:18	11:09
880	68	Lv Topeka	(" ")	2:38	11:09
1011	119	Lv Manhattan	(" ")	3:45	11:55
1223	187	Ar Salina	(" ")	5:35	1:10
1223	187	Lv Salina	(" ")	5:40	1:10
2114	303	Ar Ellis	(" ")	8:20	3:05
2114	303	Lv Ellis	(Mt. time)	7:50	2:15
4280	463	Lv Cheyenne Wells	(" ")	11:49	—
5188	640	Ar Denver	(" ")	3:55	8:00

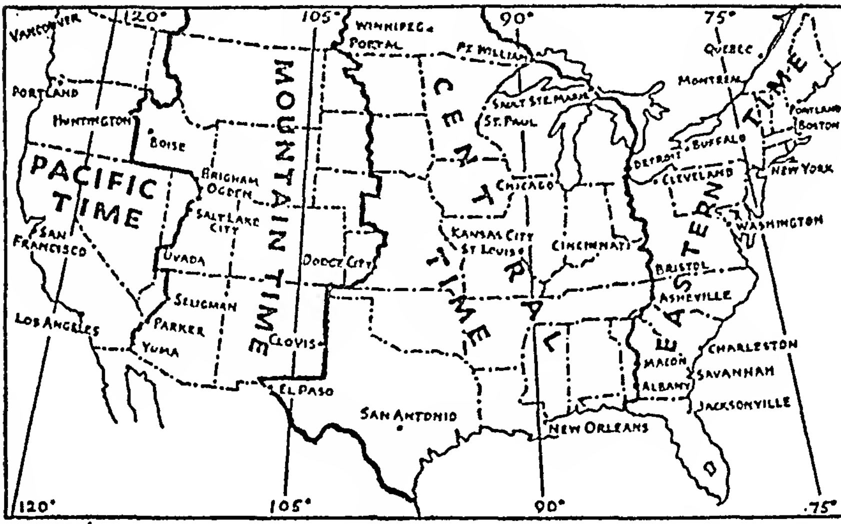
EXAMPLE

From the foregoing schedule, find:

(a) how long the trip from Kansas City to Denver takes by rail.

MILEAGES ON LONG ISLAND	Brooklyn (Prospect Park)	Flushing	Greenport	Hempstead	Jamaica	Montauk Pt.	New York (Columbus Circle)	Oyster Bay	Riverhead
Babylon	37	31	70	18	28	91	39	23	47
Brooklyn (Prospect Park)		13	99	20	10	125	7	31	76
Farmingdale	30	23	74	10	20	97	31	15	51
Flushing	13		92	13	5	118	9	20	69
Glen Cove	28	17	80	13	18	106	26	7	57
Greenport	99	92		83	89	35	100	75	23
Hempstead	20	13	83		10	107	21	15	60
Huntington	35	26	68	19	26	94	35	8	45
Jamaica	10	5	89	10		115	11	21	66
Jones Beach	29	27	85	12	22	106	33	23	62
Long Beach	23	21	92	9	16	114	27	24	69
Mineola	20	13	81	3	10	107	21	15	58
Montauk Pt.	125	118	35	107	115		126	101	49
New York (Columbus Circle)	7	9	100	21	11	126		29	77
Oyster Bay	31	20	75	15	21	101	29		52
Patchogue	55	48	50	36	46	72	57	40	27
Port Jefferson	58	51	43	42	49	70	60	35	22
Port Washington	23	12	85	10	13	111	21	16	62
Riverhead	76	69	23	60	66	49	77	52	
Rockville Centre	17	15	86	4	10	110	21	19	63
Sag Harbor	103	96	8	86	94	27	105	80	28
Southampton	92	85	19	75	83	33	94	69	17
Valley Stream	14	12	88	6	7	113	18	21	65

distances are indicated by a mileage chart, which must not be interpreted "geometrically", but rather "arithmetically"; i.e., the line segments connecting the various points are not drawn to scale, and are only diagrammatic. They have the advantage, however, of showing graphically the comparison between alternative routes.



longitude, and time within any given belt is arbitrarily considered to be everywhere the same. As will be seen from the map, the first belt, called Eastern Time, takes its time from the 75th meridian; the next, Central Time, from the 90th; the next, Mountain Time, from the 105th; and the last, Pacific Time, from the 120th. Thus there is a difference in time of 1 hour between two adjacent time belts, so that when it is 9 A.M. by Eastern Time it is 8 A.M. by Central Time, 7 A.M. by Mountain Time, and 6 A.M. by Pacific Time. Boundary lines between the time belts are irregular in order that the railroads may change their time at important cities and terminals.

Mileage between Cities

In calculating the distance between two points, when you are traveling by bus or in your own car, it is customary to consider the distance separating the two points as the actual highway mileage between them, and not the distance by rail or "as the crow flies." Sometimes tables of these distances are available as, for example, the one given here.* Occasionally

*By courtesy of the General Drafting Co. and the Colonial Beacon Oil Co. of New York.

air travel; and (3) slight inconvenience (chiefly in time lost) in getting to and from airports.

Typical Time and Rates

Air travel is rapidly becoming almost as commonplace as traveling by rail or bus. One air line company alone schedules 38 regular daily flights between New York and Washington, 19 each way; the fare one way is $12.20; round trip, $21.90.

We show a condensed schedule of another air line:

NEW YORK	Northbound			MONTREAL	
	Trip A	*Trip B*	*Trip C*	*O.W. Fare*	*R.T. Fare*
Lv New York	7:00	10:30	5:00	$ 7.95	$14.30
Ar Albany	7:58	11:28			
Lv Albany	8:05	11:35	NON-	12.35	22.20
Ar Glens Falls	——	11:57	STOP		
Lv Glens Falls	——	12:05		16.95	30.50
Ar Burlington	9:00	12:48			
Lv Burlington	9:10	12:58		19.95	35.90
Ar Montreal	9:45	1:33	7:10		

For greater distances, we give another table:

One Way Fares Elapsed Time in hrs. & min.	*Chicago*	*Los Angeles*	*Miami*	*St. Louis*
Boston	$ 51.42 5:03	$156.42 15:15	$ 83.70 8:47	$ 63.37 8:01
Detroit	$ 13.75 1:38	$118.75 13:03	$ 83.65 10:40	$ 26.20 8:23
New York	$ 44.95 3:33	$149.95 13:45	$ 71.75 8:25	$ 53.50 5:40
San Francisco	$105.00 11:12	$ 18.95 2:00	$168.20 14.30	$105.00 14:00

TRAVELING BY BOAT

Advantages

Traveling by steamship, while slower than by rail and by bus, is, to many people, the most comfortable (unless, of course, subject to seasickness). It is restful and pleasant; and there is more freedom than on a train, bus, or plane. But there is little, if any, scenic enjoyment.

Costs

Typical fares are again given without meals or sleeping accommodations, which are separate, as in the case of Pullman service on trains. We give an example of travel from New York to Boston by boat:

TYPICAL FARES

	One Way	*Round Trip*
Regular Fare	$5.50	$9.00
Special Summer Rate	4.00	6.75

TYPICAL STATEROOM PRICES

Inside rooms; two berths	(1 lower, 1 upper).....	$1.00
Semi-outside rooms; 2 berths	(" ").....	2.00
Outside rooms " "	(" ").....	3.00–3.50
Semi-outside rooms " "	(" "); lavatory	4.00
Outside rooms " "	(" "); "	5.00
Room with double bed; lavatory	....	6.00
Room with twin beds; lavatory; tub; shower	....	8.00–10.00

TRAVELING BY PLANE

Advantages

The obvious and outstanding advantage of air travel is the tremendous amount of time saved. The disadvantages include: (1) somewhat greater risk, despite the excellent record of all the air lines; (2) relatively greater costs, although these have been greatly reduced since the inception of large-scale

CHAPTER XV

MATHEMATICS OF GAMES OF CHANCE

MANKIND has always indulged in recreations and leisurely pursuits. Games and sports were apparently as intriguing hundreds of years ago as they are today. Some games, like chess and checkers, involve chiefly the skill of the players and virtually no element of "chance". In other games, like dice and roulette, chance plays the dominant if not the only role, while in still others, like poker and bridge, both skill and luck must be considered, which adds to their fascination.

In the following pages we shall explain briefly some elementary principles of mathematical probability, indicating their application to certain games of general interest. The theory of mathematical probability also has more "serious" applications; it is indispensable, for example, in the practice of insurance, in the solution of many engineering problems, and in the work of chemists, physicists, biologists, and others.

THEORY OF CHANCE

Mathematical Probability

The mathematical concept of chance, or probability, must not be confused with the psychological notion of likelihood.

Comparing air travel with rail travel between New York and Chicago, therefore, the following would be found:

N.Y. to Chicago	*By train*	*By plane*
Fare—one way	\$15	\$45
Time—one way	17 hr.	3½ hr.

Or, ratio of costs, 1 : 3, ratio of time, 5 : 1. We give finally, for general interest, the following comparison of a short trip, viz., from New York to Boston, by all four modes of travel:

	By rail	*By bus*	*By boat*	*By plane*
Approx. Fare, Round trip	\$9	\$6	\$9	\$18
Approx. Time, One way	4½ hr.	6½ hr.	15 hr.	1½ hr.

Similarly, an ace may be drawn from an ordinary deck of 52 cards in any one of 4 possible ways; a spade in any one of 13 possible ways; a face card (A, K, Q, or J) in any one of 16 possible ways; a red card in any one of 26 possible ways; etc.

Combination of Choices

If one act can be performed in 3 different ways, and then another, independent act, can be performed in 4 different ways, the two acts can be done in 3×4, or 12 different ways in all. For example, if you can enter a building by any one of 3 doors, A, B and C, and then leave the building by any one of four different other doors, a, b, c and d, you can enter the building by one door and leave by a different one in the 12 following ways, no two of which combinations are alike:

Aa	Ac	Ba	Bc	Ca	Cc
Ab	Ad	Bb	Bd	Cb	Cd

EXAMPLES

1. A woman has 3 coats and 5 hats. In how many ways can she dress, not wearing the same hat twice with the same coat?

   A coat can be selected in 3 ways,
   and then a hat may be selected in 5 ways; in all,
   $3 \times 5 = 15$ ways of dressing, *Ans.*

2. In presenting 4 men to 6 women, how many introductions are made?

   $4 \times 6 = 24$, *Ans.*

3. In how many ways may an Ace and a King be drawn from a deck of cards?

   $4 \times 4 = 16$, *Ans.*

4. In how many ways may 3 letters be posted in 4 letter boxes?
   The first letter may be dropped in any one of 4 boxes; the second letter may be dropped in any one of 4 boxes; so can the third. Hence,

   $4 \times 4 \times 4 = 64$ ways, *Ans.*

If a coin upon being tossed six times in succession has come up "heads" each of the six times, we may be impelled to feel that upon the seventh toss it is "more likely" to turn "tails" in view of the six previous heads; but this is only an emotional reaction and not a mathematical probability. Mathematically, as we shall see, on *any single throw,* the chances are even for heads or tails, irrespective of what the previous trials may have been. In the long run, the greater the number of trials, the more nearly equal will become the number of heads and tails.

Possible Ways

Before discussing *probable* events, let us say a word about *possible* events. If a coin is flipped at random, there are only *two* possible outcomes—heads or tails. Since a die has six sides, there are *six* possible ways in which it can come face up: 1, 2, 3, 4, 5 or 6, but there is only one way for a "1" to show, only one way for a "2", etc. We find, too, that there are three ways for an *even number* to show; and three ways for an *odd number* to show. If two dice are thrown, the number of possible ways in which any total may be shown by the combination of spots on the two dice follows:

Sum of Spots Shown	*Number of Ways*
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1
	36 maximum number of different ways in which two dice may fall.

POKER AND BRIDGE

Probabilities in Poker

With a 52-card deck there are 2,598,960 possible poker hands that may be dealt to a player. Table I shows a classification of these possible poker hands:

TABLE I. POSSIBLE POKER HANDS IN A 52-CARD DECK

	Actual No. Possible	*Expected No. in 10,000 Deals*	*Approximate Number of Times*
Straight flush	40	⅙	Once in 64,974 deals
Four of a kind	624	2½	" " 4,165 "
Full house	3,744	14	" " 694 "
Flush	5,108	20	" " 509 "
Straight	10,200	39	" " 256 "
Three of a kind	54,912	211	" " 48 "
Two pairs	123,552	475	" " 21 "
One pair	1,098,240	4226	" " 2½ "
No pair	1,302,540	5012	" " 2 "
Total	2,598,960	10,000	

These figures, as such, are of no great value, although the second and third columns are useful for reference. Of greater importance are the probabilities of holding various hands, as shown in Table II; these *ratios* are derived *from* Table I.

For the sake of poker enthusiasts we also show a few highly significant and obviously useful probabilities involved in "drawing" cards. Thus:

When drawing 3 cards to a pair,
the odds against making any improvement are 2½ to 1
" " " " two pairs are 5 to 1
" " " " 3 of a kind are 8 to 1
" " " " a full house are 97 to 1
" " " " 4 of a kind are 359 to 1

Definition of Probability

If a certain event can happen in p ways and fail in q ways, the probability that it will happen is $\frac{p}{p+q}$ and the chance that it will fail is $\frac{q}{p+q}$. For example, if a coin is tossed, it can come up heads in only one way, and fail to come up heads (i.e., come up tails) in only one way. The chance, therefore, of heads is $\frac{1}{2}$, and the chance of "not heads" (i.e., tails) is also $\frac{1}{2}$. That it is bound either to happen *or* not happen (i.e., fail) is certain; or, $\frac{1}{2}+\frac{1}{2}=1$. That is, the probability that an even will happen + the probability that it will fail equals unity; or $\frac{p}{p+q}+\frac{q}{p+q}=1$.

Again, on throwing a single die once, the chance of throwing a "1" is found as follows: "1" can come up in only one way, but can fail to come up in 5 ways; hence the chance of throwing a "1" is $\frac{1}{6}$, and the chance of *not* throwing a "1" is $\frac{5}{6}$. Or the chance of failing to throw a "1" is 5 times as great as the chance of succeeding. By this principle, the chance of drawing combinations of cards, etc., can be computed.

EXAMPLES

(1) What are the chances of throwing a total of seven on one throw of a pair of dice?

When two dice are thrown, the first may come up in 6 ways, and the second also in 6 ways; the two together can therefore come up in 36 different ways. As we have already seen, a total of 7 "can be shown" in 6 different ways.

Hence the probability of "throwing a 7" is $6 \div 36$, or $\frac{1}{6}$; i.e., 1 chance in 6, *Ans.*

(2) What are the chances of throwing a total of 10 on a single throw of a pair of dice?

A total of 10 can be shown in 3 different ways; hence the chance of throwing 10 is

$\frac{3}{36}$, or $\frac{1}{12}$, *Ans.*

Or again: when drawing 2 cards to 3 of a kind,

the odds against making any improvement are 8½ to 1
" " " " a full house are 15⅓ to 1
" " " " 4 of a kind are 22½ to 1

Probabilities in Bridge

When four people play bridge, a pack of 52 cards may be dealt to the players in approximately 1,000,000,000,000,000,000,000,000,000,000 different ways—a number so enormous that it can scarcely be comprehended, and even then it is nearly worthless. Devotees of this intriguing card game will doubtless be more interested in Tables I and II:

TABLE II. PRACTICAL PROBABILITIES WHEN DECLARER AND DUMMY TOGETHER HOLD 7 CARDS OF ONE SUIT

(1) Three leads will clear the suit 36% of the time.
(2) Four leads will clear the suit 84% of the time.
(3) If one honor is out:
  (a) It will fall in two leads 18% of the time.
  (b) It will fall in three leads 54% of the time.
(4) If two honors are out:
  (a) They will split 50% of the time.
  (b) They will be alone in one hand 4% of the time.
  (c) One will fall on the first lead 8% of the time.
  (d) They will both fall in three leads 50% of the time.

TABLE II. CHANCE OF HOLDING ANY PARTICULAR HAND OR BETTER IN FIRST FIVE CARDS

		Approximate Number of times	Exact Chance
Any pair	or better	Once in 2 deals	.4988
Pair of Jacks	" "	" " 5 "	.2062
Pair of Queens	" "	" " 6 "	.1737
Pair of Kings	" "	" " 7 "	.1412
Pair of Aces	" "	" " 9 "	.1087
Two pairs	" "	" " 13 "	.0762
Three of a kind	" "	" " 35 "	.0287
Straight	" "	" " 132 "	.0076
Flush	" "	" " 270 "	.0037
Full house	" "	" " 588 "	.0017

TABLE I. CHANCES OF BEING DEALT ANY SPECIFIED NUMBER OF HONORS

Number of Honors	*Frequency*
13	1 in 10,000,000
12	6 in 1,000,000
11	1 in 10,000
10	1 in 1,000
9	1 in 100
8	4 in 100
7	11 in 100
6	20 in 100
5	26 in 100
4	21 in 100
3	12 in 100
2	4 in 100
1	7 in 1,000
0	5 in 10,000

tions). A subscriber may have an *individual line,* or he may be one of two or more subscribers on a *party line.* Party-line service is offered at a cheaper rate, because of the inconvenience entailed in being able to use only one telephone on the line at any one time. The rates for business telephones are higher than for residential telephones.

Typical Rates

Schedules of telephone rates and service charges vary somewhat from place to place, but the rates shown below are generally typical:

Class of Service	*Number of Free Local Calls Allowed per Month*	*Monthly Charge*	*Rates for Additional Local Messages—All Services*
Residential: Individual line	not more than 66	$4.25	Up to 300....5¢ each; Next 300.....4½¢ each; Next 300.....4¢ each; All others....3¾¢ each
Residential: Two-party service	not more than 57	$3.50	
Business: Individual line	not more than 75	$6.00	

EXAMPLES

1. Mr. Parker had 82 local calls in a certain month, and, in addition, toll charges amounting to 85¢. What was his total bill for the month, exclusive of Federal taxes, if he has a residential individual line?

    82—66=16 extra local calls
    Monthly charge =$4.25
    Extra local calls (16×5¢)= .80
    Toll charges = .85
    Total $5.90, *Ans.*

CHAPTER XVI

MATHEMATICS OF COMMUNICATION

We live in an age of unprecedented facilities for communicating with one another, for travel, and for sending money or merchandise from one place to another. Advances in the last two or three generations, since the development of the steam railroad, have seen a rapid succession of inventions and improvements that have made the world small indeed—the telegraph, the telephone, the marine cable, the radio, the airplane, the streamlined Diesel-motored train, the transcontinental motor bus, and long-distance haulage by motor truck have brought people and places closer together than ever before.

TELEPHONE AND TELEGRAPH

Telephone Service

Both in business activities and in personal relations the network of telephone lines crisscrossing the country is a vital part of the life of America. The growth of the telephone system has been both steady and phenomenal.

A person renting telephone service from the company is technically known as a *subscriber.* Telephone service is of three kinds: *residential, business,* and *public service* (pay sta-

Base rate		=$1.05
Overtime	(5×40¢)	= 2.00
Total		=$3.05, *Ans.* (a)

Day Rate		*Night Rate*	
Base rate	=$1.25	Base rate	=$ .70
Overtime (7×45¢)	= 3.15	Overtime (7×30¢)	= 2.10
Total	=$4.40	Total	=$2.80

$4.40—$2.80=$1.60, saving, *Ans.* (b)

Telegrams

Messages sent by telegram have the advantage that a permanent record is automatically made of the text. The cost of sending a telegram depends upon the type of service and the distance transmitted. The three most commonly used services, together with typical rates, are shown in the following table:

TELEGRAM RATES

Class of Service	*Basic Rate (according to distance)*	*Additional Charge*	*When accepted*	*When delivered*
Regular telegram	First 10 words from 20¢ to $1.20	Each add. word from 1¢ to 8½¢	Any hour of day or night	Within a few minutes
Day letter	First 50 words from 30¢ to $1.80	Each add. 10-word group from 6¢ to 36¢	Any time	Somewhat longer than a few minutes
Night letter	First 50 words from 20¢ to $1.20	Each add. 10-word group from 4¢ to 24¢	Up to 2 A. M. of day of delivery	Next morning

2. A business firm made 1268 local telephone calls during the month of December. What was the amount of the bill?

   300— 75=225
   1268—900=368

Monthly charge			=\$ 6.00
225 calls	@ 5 ¢	=	11.25
300 "	@ 4½¢	=	13.50
300 "	@ 4 ¢	=	12.00
368 "	@ 3¾¢	=	13.80
Total			\$56.55, *Ans.*

Classification of Calls

A local call is a call to another telephone within the local area. Calls to nearby areas, but outside the local area, are called suburban calls or toll calls. Long-distance calls, further away than toll calls, are classified as station-to-station and person-to-person calls.

EXAMPLES

1. On a particular long-distance person-to-person call the rate is \$1.40 for the first 3 minutes and 45¢ per minute overtime. What is the total cost of a 7-minute call? a 10-minute call?

   (a) \$1.40+4×45¢=\$1.40+\$1.80=\$3.20, *Ans.*
   (b) \$1.40+7×45¢=\$1.40+\$3.15=\$4.55, *Ans.*

2. A person wishes to make a call to a certain city where the are as follows:

   Station-to-station day rate:
   \$1.25 for 3 min. and 45¢ for each additional minute
   Evening rate:
   \$1.05 for 3 min. and 40¢ for each additional minute
   Night rate:
   70¢ for 3 min. and 30¢ for each additional minute

   Find (a) the cost of an eight-minute conversation at the evening rate; and (b) how much is saved on a ten-minute call if made at the night rate instead of the day rate?

Parcel Post

The cost of sending packages by parcel post depends upon the weight of the package and the distance it is to be sent. For this purpose, the rates are given in "zones", as shown by the following table:

PARCEL POST RATES			
Zone	*Distance*	*First Pound*	*Additional Pounds*
Local	1 to 50 miles	7¢	1¢ for each lb.
2	50 – 150 "	8¢	2¢ " " "
3	150 – 300 "	9¢	2¢ " " "
4	300 – 600 "	10¢	4¢ " " "
5	600 – 1000 "	11¢	6¢ " " "
6	1000 – 1400 "	12¢	7¢ " " "
7	1400 – 1800 "	14¢	9¢ " " "
8	over – 1800 "	15¢	11¢ " " "

Parcels may be insured at the following rates:

SCHEDULE OF INSURANCE RATES	
Valuation	*Fee*
Up to $5	5¢
Up to $25	10¢
Up to $50	15¢
Up to $100	25¢
Up to $150	30¢
Up to $200	35¢

Postal Money Orders

Money can be safely and conveniently sent "by mail," using postal money orders instead of actual cash. The cost is small,

Other Services

Many other services are also available. Holiday greetings and social telegrams have become popular; flowers may be sent by telegraph; so may travelers' checks and cash.

EXAMPLES

1. What is the cost of sending an 18-word telegram when the rate is 42—2½ (i.e., 42¢ for the first 10 words, and 2½¢ for each additional word)?

first 10 words	$.42
next 8 words @ 2½¢	.20
	$.62, *Ans.*

2. What is the cost of a 73-word day letter if the rate is 66—17 (i.e., 66¢ for the first 50 words, and 17¢ for each additional 10 words or less)?

first 50 words	$ .66
next 10 "	.17
next 10 "	.17
next 3 "	.17
	$1.17, *Ans.*

POSTAL SERVICES

Types of Delivery

There are four principal classes of mail delivery in the United States:

First Class: letters and postcards.
Second Class: newspapers, magazines, and periodicals.
Third Class: circulars and other printed matter not classified with 1st or 2nd class mail.
Fourth Class (parcel post): packages not exceeding 70 pounds in weight or 100 inches in length and girth combined.

EXAMPLES

1. Mr. Hamley wishes to mail a package weighing 14 lb. 6 oz. to a city 750 miles from his home. If he insures it for $100, what postage must he pay?

   750 miles classed as "Zone 5."

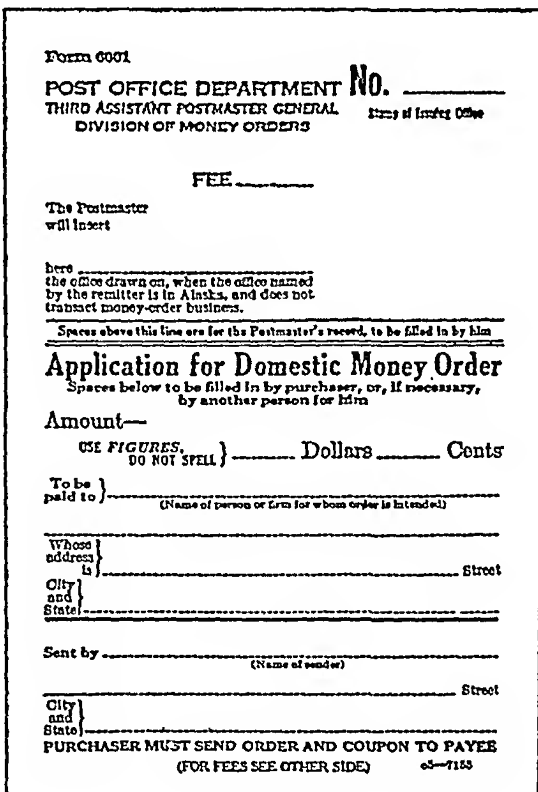
First pound	$ .11
Next 14 lb. (fraction counted as 1 lb.) @ 6¢	.84
Insurance	.25
	$1.20, *Ans.*

2. Mrs. Jackson wants to send $275 by postal money order. What fee will she have to pay?

$100	$.22
$100	.22
$ 75	.20
$275	$.64, *Ans.*

as may be seen from the following table of rates. Any amount of money may be sent; while no single money order may be written for more than $100, as many separate money orders as desired may be sent at a time.

For Orders	From	$0.01 to	$2.50......	6 cents.
	From	$2.51 to	$5.00......	8 cents.
	From	$5.01 to	$10.00......	11 cents.
	From	$10.01 to	$20.00......	13 cents.
	From	$20.01 to	$40.00......	15 cents.
	From	$40.01 to	$60.00......	18 cents.
	From	$60.01 to	$80.00......	20 cents.
	From	$80.01 to	$100.00......	22 cents.


Form 6001

POST OFFICE DEPARTMENT No. ________
THIRD ASSISTANT POSTMASTER GENERAL
DIVISION OF MONEY ORDERS

FEE ______

The Postmaster will insert here ______ the office drawn on, when the office named by the remitter is in Alaska, and does not transact money-order business.

Spaces above this line are for the Postmaster's record, to be filled in by him

Application for Domestic Money Order

Spaces below to be filled in by purchaser, or, if necessary, by another person for him

Amount—

USE FIGURES, DO NOT SPELL } ______ Dollars ______ Cents

To be paid to } ______ (Name of person or firm for whom order is intended)

Whose address is } ______ Street

City and State ______

Sent by ______ (Name of sender)

______ Street

City and State ______

PURCHASER MUST SEND ORDER AND COUPON TO PAYEE

(FOR FEES SEE OTHER SIDE)

list price, or wholesale price, is what the shopkeeper has to pay for the merchandise. He then sells it to the consumer at a retail price somewhat higher than that which he paid for it. The list price, however, is not necessarily final; in many cases it is subject to a *trade discount.*

Trade Discount

The trade discount allowed by the manufacturer, the producer, the wholesaler, or the jobber is expressed as a per cent. This per cent, applied to the list price, is the amount the purchaser is permitted to deduct from the list price. The difference is what he actually pays for it, and is known as the *net price;* hence the expressions "20% off" list price, or "subject to a trade discount of 25%," etc.

Trade discounts are offered for a variety of reasons. They make price lists flexible, allowing for frequent revision (up or down) as market conditions may necessitate, without the need of reprinting expensive catalogs; they facilitate competition; they facilitate selling; and are useful in other ways.

EXAMPLES

1. A retail gift shop bought lamps listed at $8.50 each, less 20% off. What was the amount of the discount allowed, and what was the net price?

   $8.50×20%=$1.70, amount of discount, *Ans.*
   $8.50—$1.70=$6.80, net price, *Ans.*

2. A druggist bought some brushes listed at $7.20 a dozen, subject to a trade discount of 25%. What was the net cost of each brush?

   $7.20×.25=$1.80, discount per doz.
   $7.20—$1.80=$5.40, net price per doz.
   $5.40÷12=$.45, net cost per brush, *Ans.*

   or

   $7.20÷12=$.60
   $.60×.25=$.15
   $.60—$.15=$.45

CHAPTER XVII

MATHEMATICS OF BUSINESS: BUYING AND SELLING

Among primitive peoples the exchange of goods, whether *necessities or luxuries, was carried on by simple barter.* Today, because of a tremendous variety of products as well as a highly developed industrial economy, the production and distribution of goods is far more complex. Modern business is based, among other things, upon the concept of a fair profit. In industry, raw materials are assembled and fabricated into finished products to be sold at a profit. In commerce, various commodities and semi-finished goods are bought by retail merchants and shopkeepers, who, in turn, sell them to the ultimate consumer, also in the expectation of a reasonable profit.

In this chapter we shall study the mathematical principles of buying and selling: invoices, cash discounts, trade discounts, buying expenses, inventories, and the cost of goods sold.

TRADE DISCOUNT

Catalogs and List Prices

Manufacturers and wholesalers generally issue catalogs or price sheets giving not only a description of the merchandise,—specifications, quality, sizes, etc.,—but also the list price. This

or

$\$12 \times .90 = \$10.80$, "first" net price
$\$10.80 \times .80 = \$8.64$, *Ans.*

A little study will show that when there are two successive discounts, the order in which they are computed is immaterial; either discount may be taken first. Such discounts are usually quoted by giving the larger one first: thus "30% and 10% off," rather than "10% and 30% off"; mathematically, however, they are equivalent.

It should also be clearly understood that successive *discount rates* cannot be *added* together to find the net price; for example, a discount of 25% and 10% is *not* equivalent to a single discount of 35%.

EXAMPLE

Which is greater, a discount of 30% and 10%, or a discount of 40%?

Suppose the list price is $20.
Then

$\$20 \times .70 = \$14$
$\$14 \times .90 = \$12.60$, net price at 30% and 10% off

and

$\$20 \times .60 = \$12$, net price at 40% off

We see, therefore, that a combined discount of two single rates is always *less* than it would be if one rate equal to their sum were used.

Another Short Cut

If we wish to find the net price of an article subject to two successive discounts, we may simplify the procedure as follows.

EXAMPLE

What is the net price of an item listed at $14.50, subject to successive discounts of 20% and 10%?

A Short Cut

If we wish to find the net price directly, without first finding the amount of the discount, we subtract the discount rate from 100%, and multiply the list price by the remainder so found.

EXAMPLE

What is the net price of a camera listed at \$10.60, with 15% discount?

100%—15%=85%
\$10.60×.85=\$9.01, net price, *Ans.*
Check: \$10.60×15%=\$1.59
\$10.60—\$1.59=\$9.01

When computing discounts or net prices, it is always advisable to check your work by using both methods; if the results agree, you may be quite certain they are correct.

Successive Discounts

As market conditions fluctuate, the price of an article listed in a catalog can be raised or lowered without changing the list price simply by increasing or decreasing the discount rate allowed. This explains why there is frequently a considerable difference between the quoted list price and the actual net price. This flexibility is further facilitated by the use of several discounts applied simultaneously to a given item. When two or more discounts are applied to an article they are called series discounts, chain discounts, or successive discounts.

EXAMPLE

What is the net cost of a pair of skates listed at \$12, subject to a discount of 20% and 10%?

100%—20%=80%
\$12×.80=\$9.60, "first" net price
100%—10%=90%
\$9.60×.90=\$8.64, net price, *Ans.*

Check: $\$160 \times 31.6\% = \$50.56$, amount of discount
$\$160 - \$50.56 = \$109.44$, net price

Quantity Discounts

When merchandise is bought in fairly large quantities, a slight reduction in price is often made. This is understandable when it is realized that the expense involved in selling 1000 cans of paint is not necessarily 500 times as great as when selling two cans.

EXAMPLES

1. Certain toilet sundries are listed at \$30 a dozen; if bought in lots of a gross or more they are subject to a discount of 5%. How much would be saved in ordering 2 gross at a time? What does the saving amount to on the cost of *one* item?

   2 gross $= 24$ dozen
   $24 \times \$30 = \$720$
   $\$720 \times 5\% = \$36$, total saving, *Ans.*
   $\$36 \div 24 = \$1.50$, saving per dozen.
   $\$1.50 \div 12 = 12\frac{1}{2}¢$, saving per item, *Ans.*

2. Index cards, bought a package at a time, are sold for 10¢ a package of 100. If a box of 1000 is bought, the price is 74¢. What per cent of discount does this amount to?

   $10 \times \$.10 = \$1.00$
   $\$1.00 - \$.74 = \$.26$, saving on \$1.00
   $\$1.00 \div \$.26 = .26 = 26\%$, discount rate required, *Ans.*

THE INVOICE

Billing: Extensions

For purposes of accurate records, whenever goods are bought or sold, an invoice, or itemized bill, is rendered. When the unit price of each item is multiplied by the number of such items, the extensions are said to have been made. A typical invoice, with extensions, is shown below:

$100\% - 20\% = 80\%$
$100\% - 10\% = 90\%$
$90\% \times 80\% = 72\%$
$\$14.50 \times .72 = \$10.44$, net price, *Ans.*

Equivalent Single Rate

The method just shown suggests how to find a *single discount rate* equivalent to the application of two or more successive discount rates. In this instance, subtracting 72% from 100% gives 28%, which is the required equivalent single rate; as already pointed out, the single equivalent rate is slightly less than the *sum* of the two separate rates.

EXAMPLE

What single discount rate is equivalent to a discount of 40% and 15%?

$100\% - 40\% = 60\%$
$100\% - 15\% = 85\%$
$60\% \times 85\% = 51\%$
$100\% - 51\% = 49\%$, equivalent single discount, *Ans.*

Occasionally an item will be offered with three and possibly more discounts. In such cases, what has already been said concerning two successive discounts still holds true, viz.:

(1) The order of taking the discounts is immaterial.
(2) They cannot be added to find an equivalent single rate.
(3) Similar short cuts may be used for finding both the equivalent single rate as well as the net price.

EXAMPLE

A small machine is listed at $160, subject to a discount of 20%, 10% and 5%. Find (a) the net price, and (b) the single rate equivalent to the discount series offered.

$80\% \times 90\% \times 95\% = 68.4\%$, or .684
$\$160 \times .684 = \$109.44$, net price, *Ans.*
$100\% - 68.4\% = 31.6\%$, single rate equivalent to discount series, *Ans.*

2/10, 1/30, n/60 means "2% off if paid within 10 days; 1% if paid within 30 days; net in 60 days."

5/15 E.O.M. means "5% off for cash if paid within 15 days after the end of the month in which bill is received."

EXAMPLES

1. A bill of goods dated May 5 and amounting to $122.50 is marked "3/10, 2/30, n/60." What is the amount of the check to be sent if paid on May 12? June 10? July 1?

   (a) $122.50×3%=$3.68
   $122.50—$3.68=$118.82, *Ans.*

   (b) $122.50×2%=$2.45
   $122.50—$2.45=$120.05, *Ans.*

   (c) Full amount, $122.50, *Ans.*

2. The Modern Gift Shop received the following invoice for merchandise purchased:

   6 doz. Novelty Ash Trays @ 10.75, less 20%
   3 doz. Book Ends @ 24.80, less 25%
   4 doz. Trays @ 18.50, less 20% and 10%
   Terms, 3/10, n/30

   If paid within two days after receipt of the bill, what was the amount of the remittance?

$10.75× 6=$64.50
$64.50× .8=$51.60

$24.80× 3=$74.40
$74.40×.75=$55.80

$18.50× 4=$74
$74×.8×.9=$53.28

	$ 51.60
	55.80
	53.28
	$160.68
	×.03
	$ 4.82

$160.68, invoice
4.82, cash discount
$155.86, net amount, *Ans.*

EXCELSIOR LINENS, INC.
808 Main Street, Erie, Pa.

April 3, 19—

SOLD TO:

Triangle Stores
South Linden St.

Terms: 2/10, n/60

18 doz.	Turkish Towels @ 9.20	165	60
24 doz.	Kitchen Towels @ 7.50	180	00
10 doz.	Bath Mats @ 18.75	187	50
6 doz.	Luncheon Sets @ 22.50	135	00
		668	10

Cash Discount

Merchants are frequently allowed a discount "for cash," which means that if a bill is paid within a specified period from the time it is due, a certain stated discount will be allowed for prompt payment. It is decidedly to the *seller's* advantage to do this, since (1) it affords him ready cash which he can use immediately in his business, (2) it reduces the cost of collecting delinquent or "slow" accounts, and (3) the amount lost in uncollectible accounts is decreased. Obviously it is also to the *purchaser's* advantage to avail himself of a cash discount whenever possible.

Terms of Discount

Cash discount rates generally vary from 1% to 3%, depending upon the nature of the business and the length of time allowed for payment. The *terms* are generally stated on the bill; e.g.:

1/10, n/30 means "a cash discount of 1% allowed if paid within 10 days; if not, the full amount is due 30 days after date of bill."

3/10 or 2/60 means "3% off for cash if paid within 10 days, or 2% off if paid within 60 days."

Cost of Sales

Suppose a retail business wishes to determine the cost of the goods sold during the first quarter of the year; the procedure shown below would then be followed.

EXAMPLE

Inventory as of Jan. 1		$ 8,462
Purchases during Jan.	$3140	
" " Feb.	2785	
" " Mar.	3825	9,750
Merchandise for sale		$18,212
Inventory as of Mar. 31		6,388
Cost of Goods Sold		$11,824

COST OF GOODS SOLD

Buying Expenses

The actual cost of merchandise frequently includes the shipping charges,—whether by freight, express, or parcel post. This may be added to the invoice cost, or it may be paid by the purchaser direct. If the cost of transportation is added to the bill, the cash discount is computed on the invoice cost first; then the shipping charges are added to the net price. Allowances for damaged goods or returns are deducted from the invoice price *before* the cash discount is computed.

EXAMPLE

Gaylord Bros. purchased \$240 worth of merchandise, subject to a discount of 20%, with 2% for cash. Express charges amounted to \$4.78. They returned items amounting to \$15. What was the amount of the check sent in payment?

$240—$15=$225, list price
$225×.8=$180, net invoice price
$180×.02=$3.60, cash discount
$180—$3.60=$176.40, net price
$176.40+$4.78=$181.18, actual cost, *Ans.*

Inventories

The businessman, for various reasons, must keep a record of the value of the stock on hand. Such a record is known as an *inventory,* and is taken periodically—usually once or twice a year, sometimes oftener. The prices and extensions are generally entered on a cost basis, although sometimes the selling prices are also entered. One of the chief purposes of inventories is to enable the merchant to determine the cost of the goods sold during a particular period of time,—information which, as we shall see in the following chapter, is indispensable in determining the profit made.

the gross profit. When expressed as a per cent, the margin, or gross profit, is based on the selling price.

EXAMPLES

1. A dealer bought a radio at an invoice cost of \$24.50, with express charges amounting to \$2.75. He sold it for \$39.95. What was his margin, and per cent of margin?

\$24.50, invoice cost	\$39.95, selling price
2.75, transportation	27.25, cost
\$27.25, net cost	\$12.70, margin, *Ans.*

$$\frac{\$12.70}{\$39.95}=.318=31.8\%, \text{ per cent of margin, } \textit{Ans.}$$

2. A hardware merchant purchased a shipment of window screens listed at \$180, less 20%. Freight charges totaled \$8.40. He sold the screens for \$306; allowances for returns amounted to \$4.30. (a) What gross profit did he realize on the sale of these screens? (b) What was the per cent of gross profit?

$\$180\times.8=\$144$, net price
$\$144+\$8.40=\$152.40$, cost of sales

\$306.00=gross sales	(a) \$301.70=net sales
4.30=allowances	152.40=cost of sales
\$301.70=net sales	\$149.30=gross profit, *Ans.*

(b) $\$149.30\div\$301.70=49.5\%$, per cent of gross profit, *Ans.*

3. The Dolly Madison Milliners bought a consignment of 600 hats at \$18 a dozen. They sold 500 of the hats for \$3.95 each, and closed out the remainder at \$2 each. (a) What gross profit was realized on the lot? (b) What was the average per cent of gross profit?

$$\frac{\$18}{12}\times600=\$900, \text{ cost of goods sold}$$

$$\begin{aligned}\$3.95\times500&=\$1975\\ \$2.00\times100&=\underline{\quad200}\\ &\$2175, \text{ total sales}\end{aligned}$$

CHAPTER XVIII

MATHEMATICS OF BUSINESS: OVERHEAD AND PROFIT

The initial cost of goods is by no means the only cost assumed by the merchant or the retail store. Before the merchandise can be sold, many other expenses are entailed, such as salaries of employees, rent, the cost of advertising and displays, of packaging and delivery, of collecting accounts, and miscellaneous expenses, such as postage, telephone, electric light, supplies and equipment, insurance, taxes and depreciation. The total of these selling expenses, variously known as overhead, operating expenses, or the cost of doing business, is a sizable part of the ultimate selling price to the consumer.

In the present chapter we shall see the relationship of costs and operating expenses to profit, why the selling price is considerably higher than the wholesale cost, and how the selling price is determined.

GROSS PROFIT

Margin

The difference between the price a merchant receives for his goods and the price he paid for them is technically known as the margin of profit, or *margin* for short. It is also called

EXAMPLES

1. The Academy Book Shop's sales during November were \$1844. The operating expenses for the month amounted to \$362, and the cost of goods sold was \$1239. Find the net profit for November.

\$1844=sales	\$605=gross profit
1239=cost of sales	362=overhead
\$ 605=gross profit	\$243=net profit, *Ans.*

2. Henry Cooper operated a gasoline filling station. During a certain period his sales of gas, oil, and auto accessories totaled \$1465; the cost of goods sold was \$831.80. His operating expenses during that time were: rent, \$110; helpers' wages, \$165; electricity, \$14.75; telephone, \$8.66; supplies, \$9.10. What net profit did he realize?

Operating expenses:

\$110.00	\$1465.00=sales
165.00	831.80=cost
14.75	\$ 633.20=gross profit
8.66	307.51=overhead
9.10	\$ 325.69=net profit, *Ans.*
\$307.51	

Per Cent of Net Profit

The net profit of a retail business should always be expressed *as a per cent of the net sales.* There are a number of good reasons for this practice, all of which have to do with accurate analysis of the business. In other words, by using the selling price as a basis, it is easier to see the true relations of cost, overhead, and profit to each other, and to the selling price; this will be even more evident shortly when we discuss the determination of the selling price.

EXAMPLES

1. A stationery store's sales for a certain period were \$960. Their

(a) \$2175—\$900=\$1275, gross profit, *Ans.*
(b) \$1275÷\$2175=58.6%, per cent of gross profit, *Ans.*

Overhead

As already suggested, the cost of doing business, or overhead, includes all of the expenses incurred in merchandising or marketing goods. The following illustration is typical of many small businesses, as, for example, a "specialty shop."

EXAMPLE

During a certain month the Town Toy Shop had sales amounting to \$1682. Its expenses for the month were:

Rent	\$140.00
Electricity	16.92
Telephone	12.74
Supplies	6.25
Delivery service	24.80
Pay roll	172.00
Sundry expenses	8.13
	\$380.84

(a) What per cent of its sales was the overhead?
(b) If they did a business of \$24,500 a year, what was the annual cost of doing business?

(a) \$380.84÷\$1682=22.6%, overhead, *Ans.*
(b) \$24,500×22.6%=\$5,537, annual cost of doing business, *Ans.*

NET PROFIT

The difference between the gross profit and the operating expenses gives the net profit. If the operating expenses are greater than the gross profit, as it sometimes is, there is a net loss. This may be summarized as follows:

(1) Net Profit=Gross Profit—Operating Expenses
(2) Net Profit=Sales—(Cost+Overhead)
(3) Net Profit=Gross Profit—Overhead

2. An analysis of a retail merchant's records shows that his average operating expenses were 23% and his average net profit was 12%. In a quarterly period when his sales amounted to \$8626, what were his operating expenses? his net profit?

   $8626×23%=$1983.98, operating expenses, *Ans.*
   $8626×12%=$1035.12, net profit, *Ans.*

3. A dealer estimates his overhead to be 18% of sales, and his gross profit as 26% of sales. During a given period his sales amounted to \$24,500. Find: (a) his margin; (b) the cost of goods sold; (c) his overhead; and (d) his net profit.

   (a) $24,500×26%=$6370, margin, *Ans.*
   (b) $24,500—$6370=$18,130, cost of goods sold, *Ans.*
   (c) $24,500×18%=$4410, overhead, *Ans.*
   (d) $6370—$4410=$1960, net profit, *Ans.*

   Check: Sales=Cost+Overhead+Net Profit
   $24,500=$18,130+$4410+$1960

Financial Statement

Schedules showing the activities of a business are frequently made, at least once a year, commonly every six months or even quarterly. Such an analysis is known as a profit and loss statement, and exhibits the sales, costs, gross profit, operating expenses, and net profit earned by the business during a given period of time.

THE SELLING PRICE

Component Parts of the Selling Price

When either the actual retail selling price of merchandise, or the total volume of net sales, is taken as the basis (100%) for expressing costs, overhead, and profit, the relationships between them are best illustrated diagrammatically. Here the separate percentages (15%, 25%, and 60%) are hypothetical rather than typical of any particular line of business. These relations are of fundamental importance. It will be seen how, even when the selling price and the cost remain constant, the

gross profit was $720, and the net profit, $144. What was the per cent of gross profit? the per cent of net profit?

$\frac{\$720}{\$960} = \frac{3}{4} = 75\%$, per cent gross profit, *Ans.*

$\frac{\$144}{\$960} = \frac{3}{20} = 15\%$, per cent net profit, *Ans.*

2. The Graphic Camera Shop sold $2460 worth of goods costing $1420. The operating expenses amounted to $575. Find the per cent of gross profit; of net profit.

$2460, sales	$1040, gross profit
1420, cost	575, overhead
$1040, gross profit	$ 465, net profit

$1040÷$2460=42.3%, gross profit, *Ans.*
$ 465÷$2460=18.9%, net profit, *Ans.*

Overhead as a Percentage

Over a period of years, an established business operates at a fairly constant per cent of overhead; or at least it becomes possible to estimate rather accurately what the current operating expenses are, judged in the light of previous experience. In such cases the overhead or operating expense ratio is expressed as a per cent, again based on the selling price. This per cent can then be applied to a single item, to any department of the business, or to the business as a whole.

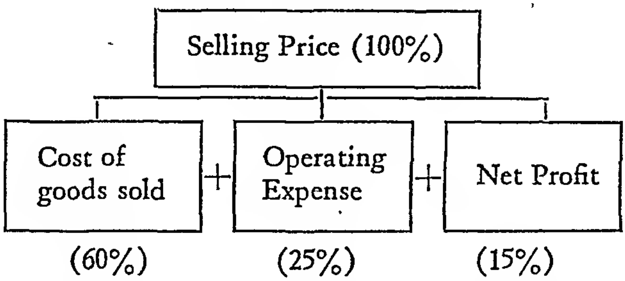
EXAMPLES

1. A sporting goods shop sells a tennis racket costing $6.50 for $10.75. If their cost of doing business is 20% of sales, what is the net profit on the sale of one racket? What is the per cent of net profit realized?

$10.75—$6.50=$4.25, gross profit
$10.75×20% =$2.15, overhead
$2.10, net profit, *Ans.*
$2.10÷$10.75=19.5%, net profit, *Ans.*

Determining the Selling Price for a Known Gross Profit

In view of these relationships, the selling price of merchandise may be determined with little or no guessing. For example, if experience shows that a certain per cent of gross profit is



essential for the successful operation of a particular line of business, then knowing the cost of the article and the required per cent of gross profit, it is a simple matter to determine the necessary selling price.

EXAMPLES

1. An electric toaster costs a hardware dealer \$4.80. At what price should he sell it to make a gross profit of 40% on the selling price?

Selling price = 100%		Cost = \$4.80
Gross profit = 40%		60% of S. P. = \$4.80
Cost = 60%		1% of S. P. = .08
		100% of S. P. = \$8.00, *Ans.*

2. A stationer buys fountain pens at \$18 a dozen. If he wishes to realize a gross profit of 25%, what should be the retail price of one fountain pen?

Selling price = 100%	Cost per doz. = \$18
Gross profit = 25%	Cost per pen = \$ 1.50
Cost = 75%	75% of S. P. = \$ 1.50
	or ¾ of S. P. = \$ 1.50
	¼ of S. P. = \$ .50
	S. P. = \$.50 × 4 = \$ 2, *Ans.*

HARRINGTON BROS., Inc.

Financial Statement, As of Sept. 30, 19—

Gross Sales		$18,400.00
Less Allowances		400.00
Net Sales		$18,000.00
Less Cost of Goods Sold:		
Inventory, July 1	$ 3,415.00	
Purchases for 3rd Quarter	13,620.00	
Goods available for sale	$17,035.00	
Less Inventory, Sept. 30	2,472.00	
Cost of Goods Sold		$14,563.00
Gross Profit on Sales		$ 3,437.00
Less Operating Expenses:		
Rent	$ 450.00	
Salaries	1300.00	
Advertising	225.00	
General Expenses	82.00	
Depreciation	110.00	
Total Operating Expenses		$ 2,167.00
Net Profit for the 3rd Quarter		$ 1,270.00

profit may be increased by decreasing the overhead, and vice versa. Or again, with a fixed selling price and rising costs, the profit may still be held constant by diminishing the overhead, if this is possible. Or once more, as the sales increase, the per cent of net profit (as well as the amount of profit) may increase, since some of the items which make up the overhead, such as rent, do not increase with larger sales, and therefore the actual total overhead becomes a smaller per cent of the total sales.

Overhead $=20\%$
Net profit$=\underline{15\%}$
35%
Cost$=100\%-35\%=65\%$
65% of S. P. $=\$13$
S. P.$=\$13\div{}^{65}/_{100}=\$20$ per doz.
$\$20\div12=\$1.67$, S. P. per necktie, *Ans.*

4. In a certain year the Franklin Company had sales amounting to \$40,000. Their annual overhead ran \$8,000. What was their per cent of overhead? If they expect to make a net profit of 8%, at what price should they sell an item that costs them \$36?

$\$8000\div\$40{,}000=20\%$, overhead, *Ans.*
$100\%-20\%-8\%=72\%$, cost of goods sold
72% of S. P.$=\$36$
S. P.$=\$36\div72\%={}^{3600}/_{72}=\$50$, *Ans.*

Mark-Up on Merchandise

The *mark-up* refers to the additional amount that must be added to the cost to provide for both the necessary overhead and the desired net profit. Thus it will be seen that the mark-up is the same as the gross profit, or margin. The *rate of mark-up,* however, is based upon the *cost,* and not upon the selling price.

EXAMPLES

1. A piece of furniture costs \$48 wholesale. At a 50% mark-up, what is the retail selling price? the per cent of gross profit?

$\$48\times50\%=\$24$, the mark-up
$\$48+\$24=\$72$, the selling price, *Ans.*
$\$24\div\$72=33\frac{1}{3}\%$, gross profit based on selling price, *Ans.*

2. A dress shop buys a certain style of dress at \$7.95 wholesale. If the mark-up is 100%, what is the selling price of the dress? the per cent of gross profit?

$\$7.95\times100\%=\$7.95$, mark-up (gross profit)
$\$7.95+\$7.95=\$15.90$, selling price, *Ans.*
$\$7.95\div\$15.90=50\%$, gross profit on selling price, *Ans.*

3. A set of drawing instruments costs \$24, less 25%. To be sold at a gross profit of 33⅓%, what retail price must be asked for it?

$$\begin{aligned} \$24\times.75 &= \$18,\text{ cost} \\ \tfrac{2}{3}\text{ of S. P.} &= \$18 \\ \tfrac{1}{3}\text{ of S. P.} &= \$\ 9 \\ \text{S.P.} = \$9\times 3 &= \$27,\ \textit{Ans.} \end{aligned}$$

Determining the Selling Price for a Desired Net Profit

Assuming that, on the basis of past experience, a merchant knows what it cost him to do business, he can determine the selling price to yield a desired per cent of net profit.

EXAMPLES

1. A heating contractor installs an oil burner unit which costs him \$66. If his selling expenses are \$18, and he wishes to realize a net profit of 20%, how much should he charge for it?

$$\begin{aligned} \text{Cost} &= \$66 \\ \text{Overhead} &= \underline{\ \ 18} \\ & \quad\ \$84 \end{aligned} \qquad \begin{aligned} \text{Net profit} &= 20\%\text{ of S. P.} \\ \text{Cost}+\text{overhead} &= 80\%\text{ of S. P.} \end{aligned}$$

$$\begin{aligned} 80\%\text{ of S. P.} &= \$84 \\ 100\%\text{ of S. P.} &= \$84\times\tfrac{5}{4} = \$105,\ \textit{Ans.} \end{aligned}$$

2. A department store paid \$420 for a shipment of novelties. If their operating expenses are 18% of sales, how much should they get for these novelties in order to realize a net profit of 12%?

$$\begin{aligned} \text{Overhead} &= 18\%\text{ of sales} \\ \text{Net profit} &= \underline{12\%}\text{ of sales} \\ & \quad 30\%\text{ of sales} \\ \text{Cost} &= 70\%\text{ of sales} \\ 70\%\text{ of sales} &= \$420 \\ \text{Sales} &= \$420\div.70 = \$600,\ \textit{Ans.} \end{aligned}$$

3. The Bandbox Haberdashery's cost of doing business is 20% of net sales. The desired net profit is 15%. At what price should they sell neckties which cost them \$13 a dozen?

It should be noted that the per cent of profit is never the same as the per cent of advance; it may be the same as the per cent of margin when and only when there are no selling expenses. Strictly speaking, the per cent of profit is always a per cent of the selling price, and not a per cent of the cost.

3. A table lamp costing \$16 is sold at a mark-up of 75%. If the overhead is 25%, what is the net profit? the per cent of net profit on the selling price?

$$\begin{array}{ll} \text{Cost} & = \$16 \\ \text{Mark-up} = 75\% \times \$16 = & \underline{\ 12} \\ \text{Selling price} & = \$28 \\ \text{Overhead} = 25\% \times \$28 = \$7 & \end{array}$$

$$\begin{array}{ll} \text{Selling price} & = \$28 \\ \text{Cost} + \text{overhead} = & \underline{\ 23} \\ \text{Net profit} & = \$\ 5, \textit{ Ans.} \end{array}$$

$$\text{Per cent net profit of S. P.} = \frac{\$5}{\$28} = 17.9\%, \textit{ Ans.}$$

Check:

Gross profit=mark-up =\$12

Gross profit=overhead+net profit
(\$12) = (\$7) + (\$5)

The mark-up is also called the *advance*. The following tabular summary shows the relations between the various terms:

Given: Cost=\$24; S. P.=\$40; Overhead=\$11		
Margin (Gross profit)	=\$16	Rate of margin=$\frac{16}{40}$=40%
Advance (Mark-up)	=\$16	Rate of advance=$\frac{16}{24}$=$66\frac{2}{3}$%
Profit (net profit)	=\$5	Rate of profit=$\frac{5}{40}$=$12\frac{1}{2}$%

By using the term "margin" instead of "gross profit," the term "profit" suffices for "net profit." By using "advance" instead of "mark-up," the *rate of advance* (based on cost) will not be confused with the *rate of margin* (based on selling price), and neither of them will be confused with the *rate of profit*.

terest, the calculation of discount on a note, the interest on personal loans, and the arithmetic of installment buying, which is a form of borrowing money.

SIMPLE INTEREST

Loans and Interest

Money is commonly being borrowed and returned in connection with business activities. Such loans are perfectly legitimate and entirely useful, if made in accordance with certain well established principles and regulations. When an individual or a business firm borrows money from another person or from a bank, *interest* is paid for the privilege of using the borrowed money for a specified period, at the end of which time the amount borrowed, called the *principal,* must be returned, and the interest due must also be paid.

Simple Interest Formula

The amount of interest due on a particular loan depends upon three factors: (1) the amount of the *principal,* (2) the length of *time* it is borrowed, and (3) the prevailing or stipulated *rate* of interest. The amount of interest is directly proportional to each of these three factors; this means that doubling the principal, or the time, or the rate, will in each case double the amount of interest, and so on. This relationship is expressed by the simple interest formula

$$I=PRT,$$

where I=interest, P=principal, R=rate (in per cent), and T=time (in years). The total amount due at the termination of the loan, i.e., principal plus interest, is called the *amount* (A); hence

$$A=P+I;$$
$$\text{and } A=P+PRT,$$
$$\text{or } A=P(1+RT).$$

These formulas are the basis of simple interest computations.

CHAPTER XIX

MATHEMATICS OF BORROWING MONEY

BOTH IN BUSINESS PRACTICE and in personal affairs, money is frequently borrowed and loaned. The privilege of using someone else's money for a period of time is paid for in the form of interest. While the taking of interest did not receive general social approval until after the Middle Ages, even the Ancients engaged in the practice, frequently charging exceedingly high rates of interest, such as 30 and 40 per cent. In the 15th century, interest was sometimes looked upon as a fine for delay in the repayment of a loan, and sometimes as compensation for the possible gains which the lender might have obtained had he used the money himself. In those early days interest was commonly referred to as *usury;* but today this term applies only to excessively high rates of interest.

The amount of a loan and the length of time for which it is extended are subject entirely to the discretion of the borrower and lender. This is not true of the rate of interest, however, which is determined, in part, by complex economic factors over which they have no control. The mathematics involved in determining the prevalent interest rate is far beyond the scope of this book.

In this chapter we shall discuss the principles of simple in-

EXAMPLE

What is the interest on a $2000 bond at 6% for 82 days?

Interest	for	60 da.=	$20.00
"	"	15 da.=	5.00
"	"	5 da.=	1.667
"	"	2 da.=	.667
			$27.334, or $27.33, *Ans.*

When the rate of interest differs from 6%, the same method is still applicable, with appropriate modification.

EXAMPLES

1. What is the interest on a loan of $800 at 4% for 100 days?

Interest	on	$800	for	60 da.	at	6%=	$ 8.00
"	"	"	"	30 da.	"	" =	4.00
"	"	"	"	10 da.	"	" =	1.333
"	"	"	"	100 da.	"	" =	$13.333
Interest	on	$800	for	100 da.	at	1%=	$2.222
"	"	"	"	" da.	"	4%=	$8.89, *Ans.*

2. A delinquent tax bill of $124 is 22 days overdue. If the fine consists of interest at 7% for the number of days overdue, what does the interest penalty amount to?

Interest	on	$124	for	60 da.	at	6%=	$1.24
"	"	"	"	20 da.	"	" =	.413
"	"	"	"	2 da.	"	" =	.041
Interest	on	$124	for	22 da.	at	6%=	$ .454
"	"	"	"	" da.	at	1%=	.076
"	"	"	"	" da.	at	7%=	$ .530
							or $ .53, *Ans.*

Finding the Principal

Occasionally we wish to find what principal sum will yield a given amount of interest in a given time and at a specified rate. Solving the formula I=PRT for P, we obtain:

$$P=\frac{I}{RT}.$$

EXAMPLES

1. What is the simple interest on \$400 for 8 months at 5%?
   $\$400 \times \frac{5}{100} \times \frac{8}{12} = \$13.33$, *Ans.*
2. Find the amount of \$1600 for 90 days at 3½%.
   $\$1600 \times \frac{7}{200} \times \frac{90}{360} = \$14$, interest
   $\$1600 + \$14 = \$1614$, amount, *Ans.*

The examples shown were solved by the so-called "cancellation method." It is also to be noted that unless otherwise specified, a year is taken as 360 days.

The "60-Day Method"

Short-term business loans are frequently made for a period of 30, 60, or 90 days. In such cases, the interest may be conveniently computed by the "60-day method":

If the annual rate is 6%, then the rate for 6 mo.=3%,
" " " 4 mo.=2%,
" " " 2 mo.=1%.

That is, at 6% a year, the interest for 2 mo., or 60 days, is 1% of the principal.

RULE: To find the interest on any sum for 60 days at 6%, simply move the decimal point two places to the left.

EXAMPLES

1. Find the interest on \$1327 for 60 days at 6%.
   Principal \$1327; move the decimal point 2 places to the left =\$13.27, interest for 60 days at 6%, *Ans.*
2. A businessman borrowed \$960 for 75 days at 6%. How much interest did he have to pay?

Interest on \$960 for 60 days at 6%	=\$ 9.60
" " " " 15 " " "	= 2.40
Interest on \$960 for 75 days	=\$12.00, *Ans.*

Sometimes the number of days is not a convenient multiple of 60 days, in which case the procedure is as shown:

SIMPLE INTEREST ON $1,000 FROM 1 DAY TO 6 MONTHS

(Based on 30-Day Month and 360-Day Year)

2½%	3%	3½%	4%	Days	4½%	5%	5½%	6%
0.069	0.083	0.097	0.111	1	0.125	0.139	0.153	0.167
0.139	0.167	0.194	0.222	2	0.250	0.278	0.306	0.333
0.208	0.250	0.292	0.333	3	0.375	0.417	0.458	0.500
0.278	0.333	0.389	0.444	4	0.500	0.556	0.611	0.667
0.347	0.417	0.486	0.556	5	0.625	0.694	0.764	0.833
0.417	0.500	0.583	0.667	6	0.750	0.833	0.917	1.000
0.486	0.583	0.681	0.778	7	0.875	0.972	1.069	1.167
0.556	0.667	0.778	0.889	8	1.000	1.111	1.222	1.333
0.625	0.750	0.875	1.000	9	1.125	1.250	1.375	1.500
0.694	0.833	0.972	1.111	10	1.250	1.389	1.528	1.667
0.764	0.917	1.069	1.222	11	1.375	1.528	1.681	1.833
0.833	1.000	1.167	1.333	12	1.500	1.667	1.833	2.000
0.903	1.083	1.264	1.444	13	1.625	1.806	1.986	2.167
0.972	1.167	1.361	1.556	14	1.750	1.944	2.139	2.333
1.042	1.250	1.458	1.667	15	1.875	2.083	2.292	2.500
1.111	1.333	1.556	1.778	16	2.000	2.222	2.444	2.667
1.181	1.417	1.653	1.889	17	2.125	2.361	2.597	2.833
1.250	1.500	1.750	2.000	18	2.250	2.500	2.750	3.000
1.319	1.583	1.847	2.111	19	2.375	2.639	2.903	3.167
1.389	1.667	1.944	2.222	20	2.500	2.778	3.056	3.333
1.458	1.750	2.042	2.333	21	2.625	2.917	3.208	3.500
1.528	1.833	2.139	2.444	22	2.750	3.056	3.361	3.667
1.597	1.917	2.236	2.556	23	2.875	3.194	3.514	3.833
1.667	2.000	2.333	2.667	24	3.000	3.333	3.667	4.000
1.736	2.083	2.431	2.778	25	3.125	3.472	3.819	4.167
1.806	2.167	2.528	2.889	26	3.250	3.611	3.972	4.333
1.875	2.250	2.625	3.000	27	3.375	3.750	4.125	4.500
1.944	2.333	2.722	3.111	28	3.500	3.889	4.278	4.667
2.014	2.417	2.819	3.222	29	3.625	4.028	4.431	4.833
2.083	2.500	2.917	3.333	30	3.750	4.167	4.583	5.000

EXAMPLE

An endowment fund yields \$390 interest semiannually at $3\frac{1}{4}\%$. What is the principal sum of the endowment?

$$P=\frac{\$390}{{}^{13}\!/_{400}\times{}^{6}\!/_{12}},$$

$$\text{or } P=\$390\times{}^{400}\!/_{13}\times{}^{12}\!/_{6}=\$24{,}000, \textit{ Ans.}$$

Finding the Rate or the Time

By using the formula differently, the rate may be found if the principal, time, and interest are known; and the time may be found if the principal, rate, and interest are known. Thus:

$$R=\frac{I}{PT}; \text{ and } \qquad T=\frac{I}{PR}.$$

EXAMPLES

1. If the quarterly interest on a \$2400 mortgage amounts to \$33, what is the rate of interest?

$$R=\frac{\$33}{\$2400\times\frac{1}{4}},$$

$$\text{or } P=33\times{}^{1}\!/_{2400}\times4={}^{11}\!/_{200}=.055=5\tfrac{1}{2}\%, \textit{ Ans.}$$

2. A corporation has cash reserves amounting to \$60,000 invested in "short term paper" at $1\frac{1}{2}\%$ interest. How long will it take to yield \$300 interest?

$$T=\frac{\$300}{\$60{,}000\times.015},$$

$$\text{or } T=\$300\div\$900=\tfrac{1}{3}\text{ yr., or 4 months, } \textit{Ans.}$$

Using an Interest Table

Wherever interest is computed frequently, as in banks, interest tables such as the one given here are frequently used to facilitate the calculations.

Accurate Interest

When simple interest is computed for the exact time on the basis of 365 days to the year instead of 360, it is then known as *accurate interest.*

EXAMPLE

Find the accurate interest on \$1000 from Jan. 10 to April 15 at 3%.

Jan.—21
Feb.—28
Mar.—31
Apr.—15
95 da., exact time

$\$1000 \times 3/100 \times 95/365 = \$7.81$, *Ans.*

Accurate interest is always *less* than ordinary interest on a given amount for the same number of days; as a matter of fact, accurate interest is $72/73$ of the ordinary interest. Accurate interest is used by the Federal Government and by Federal Reserve Banks; in general, it is used only where large sums of money are involved.

PROMISSORY NOTES

When a businessman gets a loan from the bank, he gives the bank a *promissory note* for the amount of the loan. The bank, of course, charges interest for the use of the money loaned. But the interest, being collected *in advance* (i.e., before the loan is returnable), is called *discount.* The amount of the loan is called the *face* of the note; it is the amount the borrower must return when the note falls due at *maturity.* The difference between the face of the note and the discount is called the *net proceeds,* and is the amount actually received by the borrower at the time he gets the loan. The *term of discount* means the exact number of days between the date on which the note is discounted and the date of maturity. The form below is a typical promissory note.

2½%	3%	3½%	4%	Mos.	4½%	5%	5½%	6%
2.083	2.500	2.917	3.333	1	3.750	4.167	4.583	5.000
4.167	5.000	5.833	6.667	2	7.500	8.333	9.167	10.00
6.250	7.500	8.750	10.000	3	11.25	12.500	13.750	15.00
8.333	10.00	11.667	13.333	4	15.00	16.667	18.333	20.00
10.42	12.50	14.583	16.667	5	18.75	20.833	22.917	25.00
12.50	15.00	17.500	20.000	6	22.50	25.000	27.500	30.00

EXAMPLE

Find, from the above table, the simple interest on $246 for 3 mo. 13 da. at 4½%.

Interest on $1000 for 3 mo. at 4½%=$11.25
" " " " 13 da. " " = 1.625
TOTAL =$12.875

Interest on $1 for 3 mo. 13 da.=$.012875
$.012875×246=$3.17, *Ans.*

Exact Time

Although businessmen ordinarily compute the time on the basis of 360 days to the year, or 30 days to the month, bankers use *exact time*. For example, the interest from March 2 to May 2 would cover 61 days, not 60 days; i.e., Mar., 29,+Apr., 30, +May, 2,=61 days. Bankers, however, use as a basis 360 days to the year; this is known as "ordinary interest for the exact time."

EXAMPLE

What is the ordinary interest at 4% on $600 from Aug. 13 to Nov. 10, using exact time?

Aug.—18
Sept.—30
Oct.—31
Nov.—10
89 da., exact time

$\$600 \times \frac{4}{100} \times \frac{89}{360} = \$5.93$, *Ans.*

Finding the Discount

The *term of discount* is the exact number of days from the date of the note to the date of maturity. Two possible cases arise: (1) if the note is discounted on the same date it is drawn, and (2) if the note is discounted at some later date.

EXAMPLES

CASE 1:

1. A 90-day note for \$500 is dated March 10 and is discounted the same day at 6%. Find the discount and the net proceeds.

   Date of maturity=June 8 (Mar. 21, Apr. 30, May 31, June 8)
   Term of discount=90 days
   $\$500 \times \frac{6}{100} \times \frac{90}{360} = \$7.50$, discount, *Ans.*
   $\$500 - \$7.50 = \$492.50$, net proceeds, *Ans.*

2. Mr. Jordan gave the bank his note for 2 months for \$300 on June 3, which the bank discounted the same day at 6%. Find (a) the amount of the discount, and (b) the net proceeds.

   Date of maturity=Aug. 3
   Term of discount (from June 3 to Aug. 3)

   June=27
   July =31
   Aug.= 3
   61 days

   $\$300 \times \frac{6}{100} \times \frac{61}{360} = \$3.05$, discount, *Ans.* (a)
   $\$300 - \$3.05 = \$296.95$, net proceeds, *Ans.* (b)

CASE 2:

3. A 60-day note for \$800, dated July 2, is discounted July 18 at 6%. Find the net proceeds.

   Date of maturity=Aug. 31
   Term of discount=44 days (July, 13+Aug., 31)
   $\$800 \times \frac{6}{100} \times \frac{44}{360} = \$5.87$, discount
   $\$800 - \$5.87 = \$794.13$, net proceeds, *Ans.*

Peekskill, N. Y., May 10, 19

400^{00}/_{100}$

Ninety days after date I promise to pay to the order of

Franklin Trust Co.

Four hundred and $^{00}/_{100}$ Dollars.

Value received. Frank Harris

Finding the Date of Maturity

There are two ways of expressing the term of a note—either as so-many-months or so-many-days. If a note runs for 2 or 3 months, the date of maturity is the same day of the second or third month following as the date of the note; if the date of the note is the last day of the month, the date of maturity is the last day of the second or third month following:

Date of Note	*Term of Note*	*Date of Maturity*
Jan. 5	3 mo.	Apr. 5
May 10	2 mo.	July 10
Apr. 1	3 mo.	July 1
Sep. 30	1 mo.	Oct. 31
Dec. 31	2 mo.	Feb. 28

If, on the other hand, the term of a note is given as 30 da., 60 da., or 90 da., then the date of maturity is found just as in obtaining the exact time, i.e., by counting the actual number of days, excluding the date on which the note is dated, but including the date of payment:

60-da. note dated May 10:

May (31–10)	=21
June	=30
July	= 9
	60

Date of maturity, July 9.

90-da. note dated March 1:

Mar. (31–1)	=30
Apr.	=30
May	=30
	90

Date of maturity, May 30.

INSTALLMENT PURCHASING

Consumer Credit

Buying merchandise "on time" is a way of extending credit to the consumer, i.e., the purchaser is borrowing money indirectly. Typical installment buying plans are illustrated below.

EXAMPLES

1. A piece of furniture was priced for cash at $42.30. The advertised payment plan was $2 down and $1 a week for 43 weeks. How much more does it cost when bought on the installment plan?

Down payment	$ 2	$45.00
Installment payments	43	42.30
Total	$45	$ 2.70, *Ans.*

2. Mrs. Jerome bought a vacuum cleaner by paying $2.50 down and $2 a month for 11 months. How much could she have saved by paying the cash price of $22.90?

$ 2.50	$24.50
22.00	22.90
$24.50	$ 1.60, saving, *Ans.*

3. By using the "convenient payment plan," Mr. Surrey bought a bicycle for his son. He paid $7.50 down and 15 weekly payments of $2 each. By paying cash he could have bought the bicycle for $35, less 2%. How much could he have saved by paying cash?

$ 7.50	100%—2%=98%
30.00	$35×.98=$34.30
$37.50, Installment price	$37.50—$34.30=$3.20, *Ans.*

Rate of Interest Paid on Consumer Credit

One commonly used method for determining the annual rate of interest paid when buying on the installment plan is based on the assumption that each installment payment contains principal and interest in the ratio of the starting unpaid

4. A \$900 note dated Feb. 8 runs for 3 months; it is discounted Mar. 4 at 6%. What are the net proceeds?

   Date of maturity=May 8
   Term of discount=65 days
   (Mar., 27+Apr., 30+May, 8)
   $\$900 \times 6/100 \times 65/360 = \$9.75$, discount
   $\$900 - \$9.75 = \$890.25$, net proceeds, *Ans.*

Interest-Bearing Notes

Sometimes a promissory note bears interest at a specified rate. In such cases the amount of discount is computed on the value of the note at maturity, which is greater than the face value, and which must be computed first.

EXAMPLES

1. A 90-day note for \$800, bearing interest at 5%, is dated March 3, and is discounted April 10 at 6%. Find the net proceeds.

   Date of maturity=June 1
   Term of note=90 days
   Interest on \$800 for 90 da. at 5%=\$10
   Value of note at maturity=\$810
   Term of discount=52 days
   Amount of discount $= \$810 \times 52/360 \times 6/100 = \$7.02$
   Proceeds $= \$810 - \$7.02 = \$802.98$, *Ans.*

2. A 3-month note for \$500, bearing interest at 4%, is dated Aug. 2 and is discounted Sept. 10 at 6%. Find the net proceeds.

   Date of maturity=Nov. 2
   Term of note=92 days
   Interest on \$500 for 92 da. at 4%=\$5.11
   Value of note at maturity=\$505.11
   Term of discount=53 days
   Amount of discount $= \$505.11 \times 6/100 \times 53/360 = \$4.46$
   Proceeds $= \$505.11 - \$4.46 = \$500.65$, *Ans.*

m=12; I=$42; B=$203; n=12

$$r=\frac{2\times12\times42}{203\times13}=.382, \text{ or } 38.2\%, \textit{ Ans.}$$

PERSONAL LOANS

Interest Charge

Many "personal loan companies," credit unions, and banks will loan small sums to individuals with no collateral. Such loans are usually repaid in equal installments. The amount of interest charged may be calculated in several ways. Sometimes, as in the case of credit unions, it is figured at 1% a month, which is equivalent to 12% annually; more often it varies from 2% to 3% a month; in many states the legal maximum is 3½% a month, which is equivalent to an annual interest rate of 42%.

EXAMPLES

1. A personal loan company loans $50 for 5 months, to be repaid in 5 equal installments of $10.90. What annual rate of interest is paid?

Total amount paid=5×$10.90	=$54.50
Amount of loan	= 50.00
Total interest paid	=$ 4.50

Using the formula from the previous section:
m=12; I=4.50; B=$50; n=5

$$r=\frac{2\times12\times\$4.50}{50\times6}=36\%, \text{ annual rate, } \textit{Ans.}$$

Alternate method:

Borrower has the use of	$50	for 1	month
" " " " "	$40	" "	"
" " " " "	$30	" "	"
" " " " "	$20	" "	"
" " " " "	$10	" "	"
This is equivalent to	$150	" 1	"

balance to the carrying charge. This method can be used only if all the installments are equal, which is true in the majority of cases. The procedure is fairly simple, and gives a reasonable approximation to a true interest rate. The formula used is:

$$r=\frac{2mI}{B(n+1)}$$

r =annual interest rate expressed as a decimal.

m=number of payment periods in one year; i.e., for monthly payments, m=12; for weekly payments, m=52; etc.

I =total interest, or carrying charge, in dollars.

B =unpaid balance at the beginning of the credit period, i.e., the cash price less the down payment, if any.

n =the number of payments called for, excluding the down payment.

EXAMPLES

1. A washing machine priced at $120 cash was sold on easy terms for a down payment of $39, and 10 monthly payments of $9 each. What was the annual rate of interest charged?

   Here m=12, I=$9, B=$81, and n=10

   or, $r=\frac{2\times12\times9}{81\times11}=\frac{24}{99}=24.2\%$, annual interest rate, *Ans.*

2. An auto radio is offered for $30 cash, or on time payments for an extra charge of 10%. That is, 10%, or $3, was added to the cash price; the down payment was $7, and the balance was paid in 13 weekly payments. What was the annual rate of interest?

   m=52; I=$3; B=$23; n=13

   $r=\frac{2\times52\times3}{23\times14}=96.9\%$, *Ans.*

3. The advertised retail price of a motorcycle is $340. It can be purchased in 12 equal monthly payments by paying $120 down and adding a carrying charge of $25. If paid for in cash, a discount of $17 is allowed on the retail price. What is the annual rate of interest paid?

Second method:

$300+$250+$200+$150+$100+$50=$1050,
equivalent loan for 1 month

$$R=\frac{I}{PT}=\frac{\$24.12}{\$1050\times 1/12}=27.6\%,\ \textit{Ans.}$$

Using the simple interest formula, $R=\frac{I}{PT}$,

$$R=\frac{4.50}{150\times\frac{1}{12}}=.36, \text{ or } 36\%, \text{ annual rate, } \textit{Ans.}$$

2. A bank advertises loans arranged on the following basis. If Mr. Arnold borrowed \$150 for 15 months on this plan, what annual rate is he paying?

You Borrow	*You Receive*	*You pay back per month for* 12 *mos.*	15 *mos.*
\$100	\$100	\$ 8.86	\$ 7.19
150	150	13.29	10.78
200	200	17.73	14.38
250	250	22.16	17.97
300	300	26.59	21.57

Each monthly payment of \$10.78 consists of two parts: \$10 repayment of principal, and \$.78 interest (carrying charge).

$m=12$; $I=15\times\$.78=\$11.70$; $B=\$150$; $n=15$

$$r=\frac{2\times12\times\$11.70}{150\times16}=11.7\% \text{ annually, } \textit{Ans.}$$

Alternate method:

$$\$150+\$140+\$130+\ldots+\$30+\$20+\$10=\$1200;$$

i.e., the loan is equivalent to a loan of \$1200 for 1 month. Hence,

$$R=\frac{I}{PT}=\frac{\$11.70}{1200\times\frac{1}{12}}=11.7\%, \textit{ Ans.}$$

3. A local finance company lends \$300, to be repaid in 6 monthly installments of \$54.02. Find the annual rate.

Total interest$=6\times\$4.02=\$24.12$

First method:

$$r=\frac{2mI}{B(n+1)}=\frac{2\times12\times\$24.12}{\$300\times7}=27.6\%, \textit{ Ans.}$$

annually, more often quarterly. Usually only such money as has been left on deposit during the entire "dividend period" draws interest for that period. For example, if a bank pays interest on January 1 and July 1, money should be on deposit on these dates (and remain on deposit for 6 months) in order to draw interest. However, quarterly interest periods (January 1, April 1, July 1, and October 1) are common banking practice. If money is to receive interest during any particular quarter, it must be kept on deposit from one interest date to the next. Sometimes banking practice allows money to draw interest from the date of deposit, provided it is left in the account for a stated period.

Computing the Interest

Most people do not bother to withdraw the interest on their savings account as it falls due. Instead, the bank "credits the interest" to the depositor's account, which simply means that it is added to his balance. This is an important point to which we shall refer again shortly. The manner of computing the accrued interest on a savings account is illustrated below. Remember that interest is allowed only on the *smallest* balance within any given period. No interest is allowed on fractional parts of a dollar.

EXAMPLES

1. On January 1 Betty Hall had a balance of $376.28 in her savings account. In March she withdrew $40. How much did she have to her credit on July 1, if she made no further deposits or withdrawals, the bank paying interest at 3% annually on Jan. 1 and July 1?

   The bank allows interest on $336 only, since she withdrew $40, so that the smallest sum on deposit during the 6 mo. period was $336.28; the 28¢ is disregarded.

   $\$336 \times \frac{6}{12} \times \frac{3}{100} = \$5.04$, accrued interest.

   Therefore on July 1 she will have

   $\$336.28 + \$5.04 = \$341.32$, balance on July 1.

CHAPTER XX

MATHEMATICS OF MONEY AT WORK

In the previous chapter we learned something about the principles of simple interest. This form of interest applies chiefly to short term loans—60 days, 90 days, a few months, or at most, a year or two. Money is also loaned for longer periods of time—10 years, 20 years, 50 years. It is then referred to as investment capital, and earns compound interest instead of simple interest. As we shall see, one of the important points in this connection is the assumption that interest earned on invested capital shall be reinvested (so that it starts to earn interest) the moment such interest is due and payable.

We shall begin by discussing interest on savings accounts, since this affords an excellent illustration of the theory of compound interest. From then on we shall describe the use of compound interest tables and the meaning of the present worth of money.

THE SAVINGS ACCOUNT

Interest Payments

Interest on deposits in a savings account is paid at certain stated periods. These interest payments, sometimes referred to as "dividends," may be paid in cash, or they may be added to the amount on deposit at stated intervals, sometimes semi-

NOTE: It should be noted that there are *five* conversion intervals; also that the interest for each interval may be computed as 3% for ½ a year, or at the rate of 1½% a year, or 1½% of the principal.

3. What is the compound amount of $2000 for 1 year at 4% compounded quarterly?

$2000.00	..........original principal
20.00	..........1% of $2000
2020.00	..........new principal at end of 1st quarter
20.20	..........1% of $2020
2040.20	..........new principal at end of 2nd quarter
20.40	..........1% of $2040.20
2060.60	..........new principal at end of 3rd quarter
20.61	..........1% of $2060.60
$2081.21	..........amount at end of 4th quarter.

Compound Interest Tables

Such computations are far too laborious for practical purposes. Consequently, in actual business and financial practice, compound interest tables, such as the one here shown, are used for convenience. Tables of this sort have been carefully computed by mathematical methods and calculating machines, but are based on precisely the same theory already described. In fact, we will show, for one number in the table, how this number might be obtained, or how it may be verified. Suppose we wish to know the compound amount of $1 for 3 years at 6%, compounded annually. As before:

$1.00	=original principal
.06	=first year's interest
1.06	=principal during second year
.0636	=second year's interest
1.1236	=principal during third year
.067416	=third year's interest
1.191016	=compound amount of $1 at 6% annually for 3 years.

ticularly if the number of conversion intervals is large. Note that fractional parts of a dollar are not disregarded as in the case of interest on savings accounts.

EXAMPLES

1. What is the compound interest on \$500 for 3 years, if interest is compounded annually at 4%?

Original principal	\$ 500.00
Add interest on \$500 at 4% for 1 yr.	20.00
Amount at end of 1st year	520.00
Add interest on \$520 at 4% for 1 year	20.80
Amount at end of 2nd year	540.80
Add interest on \$540.80 at 4% for 1 year	21.63
Amount at end of 3rd year	562.43
Deduct original principal	500.00
Compound interest on \$500 for 3 yr. at 4% annually	\$ 62.43

2. Find the compound interest on \$1000 for 2½ years at 3% per annum, compounded semiannually.

Original principal	\$1000.00
Add interest on \$1000 at 3% for 6 mo.	15.00
Amount at end of 1st period	1015.00
Add interest on \$1015 at 3% for 6 mo.	15.23
Amount at end of 2nd period	1030.23
Add interest on \$1030.23 at 3% for 6 mo.	15.45
Amount at end of 3rd period	1045.68
Add interest on \$1045.68 at 3% for 6 mo.	15.69
Amount at end of 4th period	1061.37
Add interest on \$1061.37 at 3% for 6 mo.	15.92
Amount at end of 5th period	1077.29
Deduct original principal	1000.00
Compound interest on \$1000 for 2½ yr. at 3% semiannually	\$ 77.29

TABLE I: COMPOUND AMOUNT OF $1 (con'd.)

N	3%	3½%	4%	4½%	5%	N
1	1.030 0000	1.035 0000	1.040 0000	1.045 0000	1.050 0000	1
2	1.060 9000	1.071 2250	1.081 6000	1.092 0250	1.102 5000	2
3	1.092 7270	1.108 7179	1.124 8640	1.141 1661	1.157 6250	3
4	1.125 5088	1.147 5230	1.169 8586	1.192 5186	1.215 5063	4
5	1.159 2741	1.187 6863	1.216 6529	1.246 1819	1.276 2816	5
6	1.194 0523	1.229 2553	1.265 3190	1.302 2601	1.340 0956	6
7	1.229 8739	1.272 2793	1.315 9318	1.360 8618	1.407 1004	7
8	1.266 7701	1.316 8090	1.368 5691	1.422 1006	1.477 4554	8
9	1.304 7732	1.362 8974	1.423 3118	1.486 0951	1.551 3282	9
10	1.343 9164	1.410 5988	1.480 2443	1.552 9694	1.628 8946	10
11	1.384 2339	1.459 9697	1.539 4541	1.622 8530	1.710 3394	11
12	1.425 7609	1.511 0687	1.601 0322	1.695 8814	1.795 8563	12
13	1.468 5337	1.563 9561	1.665 0735	1.772 1961	1.885 6491	13
14	1.512 5897	1.618 6945	1.731 6764	1.851 9449	1.979 9316	14
15	1.557 9674	1.675 3488	1.800 9435	1.935 2824	2.078 9282	15
16	1.604 7064	1.733 9860	1.872 9812	2.022 3702	2.182 8746	16
17	1.652 8476	1.794 6756	1.947 9005	2.113 3768	2.292 0183	17
18	1.702 4331	1.857 4892	2.025 8165	2.208 4788	2.406 6192	18
19	1.753 5061	1.922 5013	2.106 8492	2.307 8603	2.526 9502	19
20	1.806 1112	1.988 7889	2.191 1231	2.411 7140	2.653 2977	20
21	1.860 2946	2.059 4315	2.278 7681	2.520 2412	2.785 9626	21
22	1.916 1034	2.131 5116	2.369 9188	2.633 6520	2.925 2607	22
23	1.973 5865	2.206 1145	2.464 7155	2.752 1663	3.071 5238	23
24	2.032 7941	2.283 3285	2.563 3042	2.876 0138	3.225 0999	24
25	2.093 7780	2.363 2450	2.665 8363	3.005 4345	3.386 3549	25
26	2.156 5913	2.445 9586	2.772 4698	3.140 6790	3.555 6727	26
27	2.221 2890	2.531 5671	2.883 3686	3.282 0096	3.733 4563	27
28	2.287 9277	2.620 1720	2.998 7033	3.429 7000	3.920 1291	28
29	2.356 5655	2.711 8780	3.118 6515	3.584 0365	4.116 1356	29
30	2.427 2625	2.806 7937	3.243 3975	3.745 3181	4.321 9424	30
31	2.500 0803	2.905 0315	3.373 1334	3.913 8575	4.538 0395	31
32	2.575 0828	3.006 7076	3.508 0587	4.089 9810	4.764 9415	32
33	2.652 3352	3.111 9424	3.648 3811	4.274 0302	5.003 1885	33
34	2.731 9053	3.220 8603	3.794 3163	4.466 3615	5.253 3480	34
35	2.813 8625	3.333 5904	3.946 0890	4.667 3478	5.516 0154	35

TABLE I: COMPOUND AMOUNT OF $1

N	1%	1¼%	1½%	2%	2½%	N
1	1.010 0000	1.012 5000	1.015 0000	1.020 0000	1.025 0000	1
2	1.020 1000	1.025 1563	1.030 2250	1.040 4000	1.050 6250	2
3	1.030 3010	1.037 9707	1.045 6784	1.061 2080	1.076 8906	3
4	1.040 6040	1.050 9453	1.061 3636	1.082 4322	1.103 8129	4
5	1.051 0100	1.064 0822	1.077 2840	1.104 0808	1.131 4082	5
6	1.061 5201	1.077 3832	1.093 4433	1.126 1624	1.159 6934	6
7	1.072 1353	1.090 8505	1.109 8449	1.148 6857	1.188 6858	7
8	1.082 8567	1.104 4861	1.126 4926	1.171 6594	1.218 4029	8
9	1.093 6853	1.118 2922	1.143 3900	1.195 0926	1.248 8630	9
10	1.104 6221	1.132 2708	1.160 5408	1.218 9944	1.280 0845	10
11	1.115 6683	1.146 4242	1.177 9489	1.243 3743	1.312 0867	11
12	1.126 8250	1.160 7545	1.195 6182	1.268 2418	1.344 8888	12
13	1.138 0933	1.175 2639	1.213 5524	1.293 6066	1.378 5110	13
14	1.149 4742	1.189 9547	1.231 7557	1.319 4788	1.412 9738	14
15	1.160 9689	1.204 8292	1.250 2321	1.345 8683	1.448 2982	15
16	1.172 5786	1.219 8895	1.268 9855	1.372 7857	1.484 5056	16
17	1.184 3044	1.235 1382	1.288 0203	1.400 2414	1.521 6183	17
18	1.196 1475	1.250 5774	1.307 3406	1.428 2463	1.559 6587	18
19	1.208 1089	1.266 2096	1.326 9507	1.456 8112	1.598 6502	19
20	1.220 1900	1.282 0372	1.346 8550	1.485 9474	1.638 6164	20
21	1.232 3919	1.298 0627	1.367 0578	1.515 6663	1.679 5819	21
22	1.244 7159	1.314 2885	1.387 5637	1.545 9797	1.721 5714	22
23	1.257 1630	1.330 7171	1.408 3771	1.576 8993	1.764 6107	23
24	1.269 7346	1.347 3511	1.429 5028	1.608 4373	1.808 7260	24
25	1.282 4320	1.364 1929	1.450 9454	1.640 6060	1.853 9441	25
26	1.295 2563	1.381 2454	1.472 7095	1.673 4181	1.900 2927	26
27	1.308 2089	1.398 5109	1.494 8002	1.706 8865	1.947 8000	27
28	1.321 2910	1.415 9923	1.517 2222	1.741 0242	1.996 4950	28
29	1.334 5039	1.433 6922	1.539 9805	1.775 8447	2.046 4074	29
30	1.347 8489	1.451 6134	1.563 0802	1.811 3616	2.097 5676	30
31	1.361 3274	1.469 7585	1.586 5264	1.847 5888	2.150 0068	31
32	1.374 9407	1.488 1305	1.610 3243	1.884 5406	2.203 7569	32
33	1.388 6901	1.506 7321	1.634 4792	1.922 2314	2.258 8509	33
34	1.402 5770	1.525 5663	1.658 9964	1.960 6760	2.315 3221	34
35	1.416 6027	1.544 6359	1.683 8813	1.999 8896	2.373 2052	35

3. The compound amount of \$2000 in one year at 4% per annum, compounded quarterly=\$2081.21.

   From the table, under the 1% column, opposite 4 intervals, we find 1.0406040.

   \$2000×1.0406040=\$2081.2080, or \$2081.21

   Thus, to find interest compounded quarterly from the table, use the annual rate divided by 4, and the number of years multiplied by 4 as the number of intervals.

EXAMPLES

1. A trust fund of \$20,000 earns interest at the rate of 3% a year, compounded semiannually. What will the fund amount to in 10 years? How much interest will it have earned in that time?

   \$20,000×1.3468550 =\$26,937.10, compound amount, *Ans.*
   \$26,937.10—\$20,000=\$6,937.10, interest earned, *Ans.*

2. A university receives a bequest of \$50,000, payable in 6 years with accrued interest at 5% a year, compounded quarterly. How much will it receive at that time?

   \$50,000×1.3473511=\$67,367.56, *Ans.*

3. An insurance company invested \$25,000 of its reserves in securities paying 4% a year, compounded semiannually. How much interest will this investment yield in 5 years?

   \$25,000×1.2189944 =\$30,474.86, compound amount
   \$30,474.86—\$25,000=\$5,474.86, interest earned, *Ans.*

PRESENT WORTH

Present Value of a Single Sum

Thus far, all that we have said has been concerned with the problem of the *future* value of a sum of money at compound interest. We have seen that money at interest (capital) increases, or accumulates, as time goes on; and that to find the value of a sum at some future time, we multiply the amount in question by the appropriate accumulation factor. A sum of money not due and payable until some *future* date must be

Note that we have not dropped decimals or raised fractional parts more than a half. Also notice that since \$1 at 6% compounded annually amounts in 3 yr. to \$1.191016, then \$1000 under the same conditions would amount to 1000×\$1.191016, or \$1,191.02; or \$200 would amount to 200×\$1.191016, or \$238.20; etc.

In other words, a "compound interest table" really consists of a table of values of the compound amounts of \$1 for various periods at various rates. These values may be called "accumulation factors." To find the compound interest on any sum of money, therefore, we simply multiply that sum (the principal) by the appropriate accumulation factor; this gives us the compound amount of that sum. Subtracting the original principal from the compound amount so obtained gives us the compound interest required.

Let us verify the results obtained a little while ago (pages 306–307):

1. The compound interest on \$500 for 3 yr. at 4%, compounded annually=\$62.43.

   From Table I, under the 4% column, and opposite 3 conversion intervals, we find the figure 1.1248640.

   \$500×1.1248640=\$562.432
   \$562.43—\$500 =\$ 62.43, as before, but with considerably less labor.

2. The compound interest on \$1000 for 2½ yr. at 3% per year, compounded semiannually=\$77.29.

   From Table I, under the 1½% column, and opposite the *5th* conversion interval, we find the figure 1.0772840.

   \$1000×1.0772840=\$1077.284
   \$1077.28—\$1000 =\$77.28

   The difference of 1¢ in the previous answer, \$77.29, is due to raising and dropping decimal fractions; the result from the table is to be preferred, in all cases. Notice that when interest is compounded semiannually, we take half the annual rate and double the number of years in using the table. (See NOTE on page 307).

TABLE II: PRESENT WORTH OF $1

N.	1¼%	1½%	2%	2½%	3%	N
1	0.987 6543	0.985 2217	0.980 3922	0.975 6098	0.970 8738	1
2	0.975 4611	0.970 6617	0.961 1688	0.951 8144	0.942 5959	2
3	0.963 4183	0.956 3170	0.942 3223	0.928 5994	0.915 1417	3
4	0.951 5243	0.942 1842	0.923 8454	0.905 9506	0.888 4870	4
5	0.939 7771	0.928 2603	0.905 7308	0.883 8543	0.862 6088	5
6	0.928 1749	0.914 5422	0.887 9714	0.862 2969	0.837 4843	6
7	0.916 7159	0.901 0268	0.870 5602	0.841 2652	0.813 0915	7
8	0.905 3985	0.887 7111	0.853 4904	0.820 7466	0.789 4092	8
9	0.894 2207	0.874 5922	0.836 7553	0.800 7284	0.766 4167	9
10	0.883 1809	0.861 6672	0.820 3483	0.781 1984	0.744 0939	10
11	0.872 2775	0.848 9332	0.804 2630	0.762 1448	0.722 4213	11
12	0.861 5086	0.836 3874	0.788 4932	0.743 5559	0.701 3799	12
13	0.850 8727	0.824 0270	0.773 0325	0.725 4204	0.680 9513	13
14	0.840 3681	0.811 8493	0.757 8750	0.707 7272	0.661 1178	14
15	0.829 9932	0.799 8515	0.743 0147	0.690 4656	0.641 8619	15
16	0.819 7463	0.788 0310	0.728 4458	0.673 6249	0.623 1669	16
17	0.809 6260	0.776 3853	0.714 1626	0.657 1951	0.605 0164	17
18	0.799 6306	0.764 9116	0.700 1594	0.641 1659	0.587 3946	18
19	0.789 7587	0.756 6075	0.686 4308	0.625 5277	0.570 2860	19
20	0.780 0085	0.742 4704	0.672 9713	0.610 2709	0.553 6758	20
21	0.770 3788	0.731 4980	0.659 7758	0.595 3863	0.537 5493	21
22	0.760 8680	0.720 6876	0.646 8390	0.580 8647	0.521 8925	22
23	0.751 4745	0.710 0371	0.634 1559	0.566 6972	0.506 6917	23
24	0.742 1971	0.699 5439	0.621 7215	0.552 8754	0.491 9337	24
25	0.733 0341	0.689 2058	0.609 5309	0.539 3906	0.477 6056	25
26	0.723 9843	0.679 0205	0.597 5793	0.526 2347	0.463 6947	26
27	0.715 0463	0.668 9857	0.585 8620	0.513 3997	0.450 1891	27
28	0.706 2185	0.659 0993	0.574 3746	0.500 8778	0.437 0768	28
29	0.697 4998	0.649 3589	0.563 1123	0.488 6613	0.424 3464	29
30	0.688 8887	0.639 7624	0.552 0709	0.476 7427	0.411 9868	30
31	0.680 3839	0.630 3078	0.541 2460	0.465 1148	0.399 9871	31
32	0.671 9841	0.620 9929	0.530 6333	0.453 7705	0.388 3370	32
33	0.663 6880	0.611 8157	0.520 2287	0.442 7030	0.377 0262	33
34	0.655 4943	0.602 7741	0.510 0282	0.431 9053	0.366 0449	34
35	0.647 4018	0.593 8661	0.500 0276	0.421 3711	0.355 3834	35

worth less *today* than in the future, because a person receiving *that* amount of money today, *before it is due,* could invest it immediately and allow it to accumulate. By the time this sum of money became due and payable, it would have increased to *more than the amount then due.* Suppose Jones owes Smith \$100 which is not due until 4 years from now, and Jones offers to pay Smith today; then the sum of \$88.85 would be all that Jones ought to pay, and all that Smith could expect or be entitled to, assuming money to be worth 3%. At 3% compounded annually, \$88.85 would amount to exactly \$100 in 4 years. Therefore, the "present value" of \$100 due in 4 years (with money worth 3%) is \$88.85.

In short, just as the future value of a present sum is greater than its present value, so the present value, or present worth of a sum due later on, is less than its future value. The process of finding the present value is called "discounting the sum to its present value." This use of the word *discount* must not be confused with discount as used in connection with "bank discount," or "discount" on commercial paper.

Table of Present Values

In order to find the present value of \$100 due in 4 years with interest at 3%, all that we would have to do would be to divide \$100 by the accumulation factor for 4 yr. at 3%, which is 1.1255088; the result is \$88.85. The reason for *dividing* should be clear from the following outline:

$$\$88.85 \times 1.1255088 = \$100$$

$$\text{present value} \times \text{accumulation factor} = \text{future value}$$

$$\$100 \div 1.1255088 = \$88.85$$

$$\text{future value} \div \text{accumulation factor} = \text{present value}$$

or, in general terms, if P represents present value, F represents future value, and A represents the accumulation (or compound amount) factor, then:

$$F = P \times A, \text{ and } P = F \div A.$$

CHAPTER XXI

MATHEMATICS OF INVESTMENTS

TWO OF THE COMMONEST TYPES of investments are stocks and bonds. These are the principal, though not the only means by which corporations raise money to carry on their business. When a company wishes to increase its working capital, for example, it may issue bonds. A bond is essentially a loan, for a definite period, and at a specified rate of interest. Thus a 20-year, $4\frac{1}{2}\%$ bond for \$1000 is to all intents and purposes a negotiable promissory note, obligating the corporation which issues it to return the principal, or face value, \$1000, twenty years after the date of issue, and to pay interest at $4\frac{1}{2}\%$ at stated intervals during those twenty years to the holder of the bond. The holder of the bond need not be the original lender, since bonds change hands frequently in the open market.

In lieu of bonds, or in addition to them, the corporation may sell shares of stock which differ from bonds in several important respects. The shareholder, instead of lending the company money, actually purchases a share of the ownership of the company's assets. As a part-owner of the business, therefore, he assumes certain risks that the bond-holder does not assume. There is no guarantee that his money will ever be returned, and no stipulated interest, except in the case of "preferred" stock, and even then, bond interest must be paid out of

Now it is ever so much easier to multiply than it is to divide with numbers having many places, except when using a calculating machine; therefore, tables (like Table II) have been worked out which give the *reciprocals* of the accumulation factors. For example, $1 \div 1.1255088 = 0.8884870$; hence to find the present value of \$100, instead of performing the operation

$$\frac{\$100}{1.1255088} = \$88.85,$$

we perform the operation

$\$100 \times .8884870 = \$88.85$, which we can do easily because the *reciprocals of the accumulation factors are given directly by the table of present values.*

EXAMPLES

1. What is the present value of \$500 due 6 years hence, if money can be invested at 3% per year, compounded annually?
   Table II, under 3%, opposite 6 intervals, find 0.8374843.
   $\$500 \times 0.8374843 = \$418.74$, *Ans.*

2. What is the present cash value of a debt of \$800, payable in 8 years, if money can be invested at 5% per year, compounded semiannually?
   Table II, under 2½%, opposite 16 intervals, find 0.6736249.
   $\$800 \times 0.6736249 = \$538.90$, *Ans.*

Since the present value of a given sum due in the future is always less than that sum, all the values in a table of present values (Table II) are necessarily all decimals *less than unity,* (that is, *less than one*).

Bond Values and Prices

As already mentioned the par value of a bond is usually $1000. This is the amount on which the interest is based. Bonds, however, are bought and sold at prices which fluctuate according to the demand for them. Therefore, the price paid for a $1000-bond may be more or less than $1000, and is called its *market value*. Although the market value may change as quoted on the exchange, the par value never changes. If the market value is above $1000, the bond is being bought or sold at a *premium;* if below $1000, it is said to be bought or sold at a *discount.*

Bond Quotations

The daily prices of bonds, as dealt in on the exchanges, are quoted in the financial section of the newspaper. A quotation of "94½" means that the price of a $1000 bond is $945; a quotation of "104¾" means that the price of a $1000 bond is $1047.50; etc. In the newspaper, these quotations are given in abbreviated form; for example:

Beth Stl 3½s'52	106⅝—106¼
Penn RR 3¾s'70	97— 96⅛

means that

> Bethlehem Steel 3½% bonds due in 1952 sold for 106⅝, the high price for the day, and 106¼, the low price for the day, or between $1066.25 and $1062.50 per $1000 bond;

and that

> Pennsylvania Railroad 3¾% bonds due in 1970 sold at a high of 97, and a low of 96⅛ for the day, or between $970 and $961.25 per $1000 bond.

EXAMPLES

1. What must be paid for $2000 of Cleveland Union Terminal 5½% bonds quoted at 88¾?

   $1000 bond at 88¾ costs $887.50

   $887.50×2=$1775, *Ans.*

profits before dividends. The holder of common stock shares the fortunes of the corporation; if there is a profit, he receives a dividend; if not, there is no return on the investment.

INCOME FROM BONDS

Bond Interest

The date when repayment of the principal is due is called the *date of maturity.* The number of years from the date of issue to the date of maturity is called the *term* of the bond. The rate of interest named in the bond is called the *coupon rate.* The face value of the bond, or the principal amount of the loan, is called its *par value.* This is commonly \$1000, although bonds are also issued in larger denominations of \$5000 and \$10,000, and sometimes even in smaller denominations of \$500 and \$100.

The annual interest received from a bond is found by multiplying the face value (principal) of the bond by the rate of interest stipulated; thus a \$1000 4½% bond pays \$45 a year interest. The collection of this interest is facilitated by the use of *coupons* attached to most bonds. Since most bonds pay interest semiannually, each coupon on the above described bond would be worth \$22.50 on the date due.

EXAMPLES

1. Mr. Hewitt owns three \$1000 electric power bonds bearing a coupon rate of 3¾%, payable semiannually. How much interest does he collect from these bonds every six months?

   $\$1000 \times 3\frac{3}{4}\% = \$37.50$, annual interest per bond
   $\$37.50 \div 2 \quad = \$18.75$, semiannual interest per bond
   $\$18.75 \times 3 \quad = \$56.25$, total semiannual interest, *Ans.*

2. A railroad company issues \$500,000 in 3½% fifteen year bonds. What are the annual interest charges on these bonds? How much interest will the company have to pay in all during the life of these bonds?

   $\$500{,}000 \times 3\frac{1}{2}\% = \$\ 17{,}500$, annual interest charge, *Ans.*
   $\$\ 17{,}500 \times 15 \quad = \$262{,}500$, total interest on entire bond issue, *Ans.*

2. What is the current yield on the $3\frac{1}{4}\%$ bonds of the American Telephone & Telegraph Co. when selling at $109\frac{3}{8}$?

Annual income per bond = \$ 32.50
Market price per bond = \$1093.75
Current yield = \$32.50 ÷ \$1093.75 = .0297, or 2.97% to the nearest hundredth of 1%, *Ans.*

Approximate Yield to Maturity

The method described above for determining the rate of return on a bond investment is not too accurate if the bond is held until maturity. Let us see why. Consider a $4\frac{1}{2}\%$ bond, due July 1, 1956, which was bought on July 1, 1941, at a market price of 106. On a current yield basis the rate of return would be \$45 ÷ \$1060 = 4.25%. But if the bond is held until it matures on July 1, 1956, fifteen years later, the owner will receive only \$1000 for the bond, although he originally paid \$1060 for it. He has thus "lost" \$60 over a 15-year period; or,

Annual interest	= \$45
Average annual loss	= 4
Average annual income over 15-year period	= \$41

Just as the "loss" due to having paid a premium for the bond in the first place is spread equally over the entire 15-year period, so the price may be expected to decline gradually during that time from \$1060 to \$1000; or

$$\text{average market price} = \frac{\text{purchase price} + \text{par value}}{2};$$

$$\text{i.e., in this case, } \frac{\$1060 + \$1000}{2} = \$1030.$$

Thus the approximate yield to maturity equals \$41 ÷ \$1030 = .0398, or 3.98%. This is somewhat *less* than the current yield of 4.25%. If a bond is bought at a discount instead of a premium, the average annual *profit* is *added* to the annual income, which is then divided by the average market price paid.

2. If quoted at 108⅜, what must be paid for \$5000 of New Jersey Power and Light 4½s '60?

   \$1000 bond at 108⅜ costs \$1083.75

   \$1083.75×5=\$5418.75, *Ans.*

Approximate Rate of Return on Bond Investments

If the price paid for a bond were exactly \$1000, or its par value, then the coupon rate, or the rate of interest specified in the bond, would also be the rate of return received by the owner of the bond. Since, however, the price paid is usually above or below par, the actual rate of return on the investment is different from the coupon rate. If the bond is purchased at a premium (above par), the rate of return is *less* than the coupon rate; if bought at a discount (below par), the return on the investment is *higher* than the coupon rate.

The Current Yield of a Bond

One simple method used to find the rate of income, especially when a bond is "held" for a comparatively short period of time, is the *current yield* basis. For example, suppose a 5% bond is bought at 104. The annual interest received is \$50; the market value is \$1040. The current yield is then \$50÷\$1040= .0481, or 4.8%, to the nearest tenth of 1%; this represents the percentage received on the basis of the amount invested, regardless of other factors. Or,

$$\text{Current Yield} = \frac{\text{Annual Income}}{\text{Market Price}}$$

EXAMPLES

1. Mr. Halsey bought four \$1000 Great Northern Railway 4½% bonds at 89½. What was the current yield?

   Annual income per \$1000 bond=\$ 45

   Market price per \$1000 bond=\$ 895

   Current yield=\$45÷\$895 =.0503, or 5.0%, to the nearest tenth of 1%, *Ans.*

long period, the actual price trend, and so on. To find the yield to maturity more accurately, extensive tables must be consulted. However, for ordinary purposes, this method of finding the approximate yield to maturity is sufficiently accurate.

Bond Transactions

Bonds are bought and sold on stock exchanges through the agency of a *broker*. For his services as agent the broker receives a commission, which is known as the *brokerage* fee. In most cases, this brokerage fee amounts to \$2.50 per \$1000 bond. The fee must be paid to the broker both by the purchaser and by the seller of the bond; the broker receives \$5 for every bond.

When a person buys a bond, he must pay only the price quoted plus the brokerage fee. But when he sells a bond, the seller must pay not only the brokerage fee, but also a Federal *transfer tax* of 40¢ per \$1000 bond. These two charges, the brokerage and the transfer tax, are deducted from the quoted selling price to obtain the net proceeds.

EXAMPLES

1. Mr. Withers expects to purchase three \$1000 Illinois Central 4s '51 at 88¾. How much will the bonds cost him?

Price paid per \$1000 bond	=\$887.50
Brokerage per bond	= 2.50
Total cost per bond	=\$890.00

\$890×3=\$2670, *Ans.*

2. A bank sold five \$1000 Southern Bell Telephone & Telegraph 3¼s '62 at 106⅞. What net proceeds did they receive from the sale of these bonds?

Price received per \$1000 bond		=\$1068.75
Less brokerage	\$2.50	
Less tax	.40	
	\$2.90	2.90
Net proceeds per bond		=\$1065.85

\$1065.85×5=\$5329.25, *Ans.*

EXAMPLES

1. United Light and Railway $5\frac{1}{2}\%$ bonds bought in 1942 at $93\frac{1}{2}$ are due 10 years later in 1952. What is the approximate yield to maturity?

Par value =\$1000
Market price= 935
Discount =\$ 65
Annual income =\$55.00
Aver. annual profit = 6.50
Aver. annual income=\$61.50

Par value =\$1000
Purchase price= 935
2)\$1935
Aver. purchase price=\$ 967.50

$$\frac{\text{Average annual income}}{\text{Average purchase price}}=\frac{\$\,61.50}{\$967.50}=.0636=6.36\%,\ \textit{Ans.}$$

2. The United Light and Railway Co. also has 6% bonds, due in 1952. If these are bought in 1942 at a purchase price of $115\frac{3}{4}$, what is the approximate yield to maturity, 10 years later?

Market price=\$1157.50
Par value = 1000.00
Premium =\$ 157.50
Annual income =\$60.00
Aver. annual loss = 15.75
Aver. annual income=\$44.25

Par value =\$1000.00
Purchase price= 1157.50
2)\$2157.50
Aver. purchase price=\$1078.75

$$\text{Approx. yield to maturity}=\frac{\$44.25}{\$1078.75}=.0410=4.10\%,\ \textit{Ans.}$$

In these two examples it turns out that the first bond, with the lower coupon rate, yields the higher rate of return *in the long run;* its current yield is \$55÷\$935=5.88%, which is slightly higher than the current yield of the second bond, which is \$60÷\$1157.50=5.18%. Since the current yield may be slightly misleading, the approximate yield to maturity should also be known.

The reason that the "yield to maturity" as computed above is only *approximate* is that several other factors are involved, such as whether the unexpired term is relatively a short or a

1) the price quoted
2) the brokerage fee
3) the accrued interest

EXAMPLE

Mr. Norton purchased from his broker three \$1000 3¼% bonds, with interest dates March 1 and September 1. If they were bought on May 12 at 95⅜, what was the actual total cost?

Accrued interest from March 1 to May 12 is for 72 days (Mar., 31; Apr., 30; May, 11).

Price per bond	=\$953.75
Brokerage per bond	= 2.50
Accrued interest per bond	= 6.50
Total cost per bond	\$962.75

\$962.75×3=\$2888.25, *Ans.*

Selling Bonds

When bonds are sold, the accrued interest is *added* to the purchase price quoted, but the combined brokerage and tax are deducted from the gross amount received to find the net proceeds actually obtained.

EXAMPLE

On September 24, Mr. Petty sold \$5000 worth of 6% bonds at 106¼. If the interest dates are May 1 and November 1, what net proceeds did Mr. Petty realize from the sale of these bonds?

Accrual period amounts to 146 days (May, 31; June, 30; July, 31; Aug., 31; Sept., 23.)

Interest on \$1000 at 6% for 146 da.=\$24.33

Accrued interest per bond		=\$ 24.33
Price received per bond		= 1062.50
Gross am't. received per bond		=\$1086.83
Less brokerage	\$2.50	
Less tax	.40	
	\$2.90	2.90
Net proceeds per bond		=\$1083.93

\$1083.93×5=\$5419.65, *Ans.*

Accrued Interest

It should be carefully noted that in the two preceding examples no account was taken of the so-called *accrued interest.* As we have seen, interest on bonds is usually paid every six months—e.g., on Jan. 1 and July 1, or May 1 and Nov 1., etc. Therefore, if a bond is purchased between interest-payment dates (which is nearly always the case), the purchaser must pay the interest which has accumulated from the last interest-payment date to the date of purchase. This "accrued" interest does not really belong to the purchaser, but to the *previous owner* of the bond, who receives it at the time he sells it. Accrued interest is always computed on the basis of 360 days to the year, and at the same rate as specified in the bond. Let us see how this works. On October 4, Mr. Merrick purchased a 5% bond for $1000 from Mr. Boylan. The interest on this bond is payable January 1 and July 1. Mr. Boylan had collected the interest up to last July 1; on January 1, Mr. Merrick will collect the interest for the six months from July 1 to January 1. Since, however, Mr. Boylan owned the bond from July 1 to October 4 (a period of 95 days), he is entitled to 95 days' interest on $1000 at 5%, or $13.19. When Mr. Merrick purchased the bond on October 4, *he* was expected to pay Mr. Boylan the interest ($13.19) earned during those 95 days. On the following January 1 Mr. Merrick will collect $25 interest from July to January, of which $13.19 does not belong to him, but really to Mr. Boylan. However, Mr. Merrick has already paid this amount to Mr. Boylan at the time of purchase on October 4.

It's not as complicated as it seems; a bond is *earning* interest *every day,* but it is only *paid* twice a year. On any given day a bond is actually *worth* its market price plus all the interest that has accrued *up to that day* since the last interest date.

Buying Bonds

To find the actual amount that must be paid when purchasing a bond we must consider three things:

3. What is the quarterly dividend check on an investment of 200 shares of 6% preferred stock, par value $25?

   $\$25 \times .06 = \$1.50$, annual dividend per share
   $\$1.50 \div 4 = \$.37\frac{1}{2}$, quarterly dividend per share
   $\$.37\frac{1}{2} \times 200 = \$75$, quarterly dividend on 200 shares, *Ans.*

Stock Quotations and Brokerage

When stocks are bought and sold on the exchange, a commission is paid to the broker for his services. The brokerage fee is computed for "full lots" of 100 shares or multiples thereof, and depends upon the price at which the stock is bought or sold. The current rates of commission on the New York Stock Exchange are as follows:

Market Price of Stock	*Commission Rate per 100 Shares*	*Market Price of Stock*	*Commission Rate per 100 Shares*
From \$ 1 to \$ 1⅞	\$ 5	From \$20 to \$29⅞	\$15
" \$ 2 " \$ 2⅞	\$ 6	" \$30 " \$39⅞	\$16
" \$ 3 " \$ 3⅞	\$ 7	" \$40 " \$49⅞	\$17
" \$ 4 " \$ 4⅞	\$ 8	" \$50 " \$59⅞	\$18
" \$ 5 " \$ 5⅞	\$ 9	" \$60 " \$69⅞	\$19
" \$ 6 " \$ 6⅞	\$10	" \$70 " \$79⅞	\$20
" \$ 7 " \$ 7⅞	\$11	" \$80 " \$89⅞	\$21
" \$ 8 " \$ 8⅞	\$12	" \$90 " \$99⅞	\$22
" \$ 9 " \$ 9⅞	\$13	For each additional \$10 or	
" \$10 " \$19⅞	\$14	fraction thereof	\$1 more

If less than 100 shares are bought or sold at a time, the transaction is called an *odd lot;* in such cases the brokerage fee is computed on a different basis, a higher rate per share being charged than in the case of a standard lot.

EXAMPLES

1. What is the brokerage fee on the sale of 200 shares of stock selling at $86\frac{1}{2}$?

   Brokerage = \$21 per 100 shares
   $\$21 \times 2 = \$42$, *Ans.*

INCOME FROM STOCKS

Kinds of Stock

There are two important kinds of shares—*preferred* and *common*. Preferred stock entitles the owner to a fixed per cent of dividend, provided that the net profits are sufficient to warrant paying this dividend; it never pays more than the specified fixed per cent. The dividend on preferred stock must be paid before any dividend is paid on the common stock. If the profits are large enough to pay the dividend on the preferred stock first, then the common stock participates in a share of the profits beyond the preferred dividend requirement.

Par Value; Dividends

The *par value* of a share of stock, if any, is stated on the stock certificate; it is generally \$100, although sometimes it may be \$50, \$25, or even \$10. Many stocks have no par value. The *market value* of a stock is the price per share at which it is bought or sold; it has no direct relation to the par value. Stock dividends are customarily "declared" annually, semiannually, or quarterly. If a stock has a par value, the amount of the dividend is usually expressed as a certain per cent of the par value; if it has no par value, the dividend is expressed as a specified number of dollars, or cents, per share. Dividends on preferred stock are expressed as a per cent of the par value.

EXAMPLES

1. What is the annual dividend received from 50 shares of common stock, no par value, if the quarterly dividend is \$2.25 a share?

   $\$2.25 \times 4 = \$9$, annual dividend per share
   $\$9 \times 50 = \$450$, total annual dividend, *Ans.*

2. Find the annual dividend on 25 shares of stock, par value \$50, if the semiannual dividend is 4½%.

   $\$50 \times .045 = \$2.25$, semiannual dividend per share
   $\$2.25 \times 2 = \$4.50$, annual dividend per share
   $\$4.50 \times 25 = \$112.50$, total annual dividend, *Ans.*

Approximate Dividend Yield

Computing the rate of return on an investment in stocks is a somewhat simpler calculation than in the case of bonds. There is no "yield to maturity" with stocks; there is merely the approximate dividend yield.

EXAMPLES

1. Mr. Exeter purchased 200 shares of American Can Company, which paid an annual dividend of \$4 per share. If he paid 82½ for the stock, what was the rate of return on his investment?

   Price per share = \$82.50
   Brokerage per share = .21

   Total cost per share = \$82.71
   Annual dividend per share = \$4
   \$4 ÷ \$82.71 = .0484, or approximate rate of return = 4.84% *Ans.*

2. The quarterly dividend on Detroit Edison stock is 35¢ per share. If 200 shares of the stock are purchased at 21⅝, what is the per cent of income received?

   Price per share = \$21.625
   Brokerage per share = .15

   Total cost per share = \$21.775
   Annual dividend per share = \$.35 × 4 = \$1.40.
   \$1.40 ÷ \$21.775 = .0643, or 6.43%, *Ans.*

3. An investment trust purchased 1000 shares of Lorillard Tobacco 7% preferred stock at 155½. (a) What was the approximate dividend yield? (b) What was the quarterly income?

   (a) Price per share = \$155.50
   Brokerage per share = .28

   Total cost per share = \$155.78
   Annual dividend = 7% of par value (\$100), or \$7 per share.
   \$7 ÷ \$155.78 = .0449, or 4.49%, *Ans.*

   (b) Quarterly income $= \dfrac{\$7 \times 1000}{4} = \$1750$, *Ans.*

2. What is the brokerage on a purchase of 300 shares of stock purchased at $62\frac{3}{8}$?

   Brokerage=$19.00 per 100 shares
   $19.00×3=$57.00, *Ans.*

Buying and Selling Stocks

When stock is bought, no tax is required to be paid if the purchase is a "full lot". When selling stock, however, a Federal tax and a state tax are usually required in addition to the brokerage fee. In the following examples, taxes are disregarded, since they vary from time to time, and in various states.

EXAMPLES

1. Mr. Peabody bought 200 shares of Corn Products Refining Co. stock at $53\frac{1}{4}$. What did the stock cost Mr. Peabody?

   $53.25×200=$10,650.00, market price
   $18.00×2 = 36.00, brokerage fee
   $10,686.00, total cost, *Ans.*

2. Mr. Martin sold 300 shares of Anaconda Copper Co. stock at $27\frac{3}{8}$. What net proceeds did Mr. Martin receive from the sale of his shares?

   $$27\frac{3}{8}$×300=$8212.50, market price
   $15×3 = 45.00, brokerage
   $8167.50, net proceeds, *Ans.*

3. A businessman sold 500 shares of Commonwealth Edison stock for $23\frac{1}{2}$. If he paid $28\frac{3}{4}$ for it originally, how much did he lose on the transaction?

$$23\frac{1}{2}$×500=$11,750	$$28\frac{3}{4}$×500=$14,375
$15 ×5 = 75	$15×5 = 75
Proceeds=$11,675	Cost =$14,450

   $14,450, cost
   11,675, proceeds
   $ 2,775, loss, *Ans.*

the death of the insured, or upon some specific date stipulated in the policy. The person whose life is insured generally pays the premiums; the amount paid is called the *benefit;* and the person who receives the benefit is called the *beneficiary.*

Types of Policies

Today there are many varieties of policies available. The three basic types, however, are: (1) whole-life policies, (2) limited-payment policies, and (3) endowment policies.

In a *whole-life* policy, the insured pays a fixed premium every year as long as he lives; the benefit is payable only upon his death.

In a *limited-payment* policy, the insured pays a fixed premium for a limited number of years only, say 10, 15, 20 or 25 years; thereafter, if he is still living, his policy is "paid-up", and he pays no more premiums. The benefit, as before, is payable only at death.

In an *endowment* policy, the premiums are paid for a limited number of years, but the face value of the policy (i.e., the benefit) is payable at death, or, if the insured is still living at the expiration of the endowment period (premium-paying period), the amount of the benefit is payable to the insured instead of to the beneficiary.

Mathematical Theory

The scientific basis underlying the calculation of premiums and benefits includes three factors: (1) statistically determined mortality rates; (2) the accumulation of a reserve fund from the investment of premium payments received; and (3) the overhead expense in conducting an insurance business. The most interesting of these, in a way, is the "mortality factor". Without going into details (which would carry us far beyond elementary mathematics), the mathematical probability that a person of a given age will live one year longer can be determined, *in the aggregate,* with very great accuracy by mathe-

CHAPTER XXII

MATHEMATICS OF INSURANCE: PERSONAL SECURITY

LIFE INSURANCE is not gambling in the ordinary sense of the word. As a matter of fact, in all forms of insurance other than life insurance, the contingency or risk against which protection is sought may *never occur.* A house insured against fire may never burn; an accidental injury may never be sustained; one's property may never be stolen. But death is certain. Strictly speaking, life insurance is more aptly called life *assurance;* for what is really insured against is not death, which is certain, but the *premature death* of a wage earner, which is an uncertainty. Moreover, modern life insurance is based on very definite, although rather complicated, mathematics, in which there is no "guesswork". This is a highly specialized field, and those who are expert in it are known as *actuaries.*

LIFE INSURANCE

Like all other forms of insurance, life insurance is a co-operative arrangement by which large numbers of people can be benefited. A life insurance policy is a contract whereby the company, in return for certain stipulated payments called *premiums,* agrees to pay a specified sum of money either upon

ANNUAL PREMIUMS PER $1000 OF INSURANCE			
Age at Issue	*Whole-Life Policy*	*20-Payment Life Policy*	*20-Year Endowment Policy*
20	$14.63	$24.12	$44.69
25	16.56	26.42	44.79
30	19.10	29.25	45.20
35	22.24	32.49	45.89
40	26.67	36.74	47.48
45	32.35	41.82	49.91
50	39.72	48.06	53.61
55	49.49	56.16	59.40

EXAMPLES

1. A young man of 20 wishes to take out a 20-payment life policy for $1000. (a) What annual premium must he pay? (b) How much will he have to pay every 3 months if he decides to pay his premiums on a quarterly basis? (c) How much more per year will this insurance cost him if he waits until he is 25 years of age? (d) How much more will it cost in all if he delays?

   (a) $24.12, annual premium
   (b) $24.12×.265=$6.39, quarterly premium
   (c) $26.42—$24.12=$2.30, additional annual cost
   (d) $2.30×20=$46, total additional cost

2. At the age of 35, Mr. Thomas purchased a 20-year endowment policy for $3000, paying premiums annually. (a) What is the annual amount of his premium? (b) If the agent's commission was 20% of the first premium, how much did the agent receive?

   (a) $45.89×3=$137.67, annual premium
   (b) $137.67×.2=$27.53, agent's commission

3. What is the annual premium on a 20-payment life policy if issued for $5000 at age 30? (b) If the insured lives for more than 20 years, how much will he have paid in total premiums?

   (a) $ 29.25× 5=$146.25, annual premium
   (b) $146.25×20= $2,925, total premium payments

matical methods. When we say "in the aggregate" we are considering very large numbers of people. Such probabilities of living and dying at various ages constitute what is known as the mortality experience; the figures are based upon *actual records* of hundreds of thousands of people, kept over many years. That is why they are so accurate when applied to large numbers of cases; they are not applicable to individual cases. In other words, where large numbers of persons are insured, the payments received as premiums, together with the interest received on such payments when invested, are sufficient to meet all the obligated benefits, in the aggregate, even though particular individuals may deviate, one way or the other, from the statistical or average mortality rates. That is what we mean when we say that life insurance is really not gambling or guesswork.

Typical Premium Rates

Premiums on ordinary insurance policies are usually paid on an annual basis, although semi-annual and quarterly premium payments are quite common. The amount of the annual premium depends upon (1) the age of the insured person, and (2) upon the type of policy. The older the insured, the higher the rate. Of the three principal types mentioned, the whole-life policy carries the lowest premium for a given "age at issue"; the limited-payment life policy carries a somewhat higher premium (age for age), since there are fewer premium payments required; and the endowment policy carries a still higher premium, since not only are fewer premium payments received, but in many cases the benefit is paid before the insured's death. These relations are shown in the table of typical annual premium rates per $1000 of insurance.

When premiums are paid semiannually, the amount of each payment is generally found by multiplying the annual rate by .51 or .52; the quarterly rate is often taken as .265 times the annual rate.

EXAMPLES

1. Mr. Hobson bought a $5000 whole-life policy when he was 25 years of age. He died after having made 15 annual premium payments. How much more did the beneficiary receive than the total amount of premiums that had been paid in?

   $16.56×5=$82.80, annual premium.
   $82.80×15=$1242, total premiums paid in 15 yr.
   $5000—$1242=$3758, excess of benefit over premium cost

2. If I buy a 20-year endowment policy for $10,000 at age 30, (a) how much will I have to pay the company in premiums during the next 20 years? (b) how much will I receive when I am 50 years old, if still living? (c) how much more is this than the total amount I shall have paid in during those years?

   (a) $45.20×10×20=$9,040, total amount paid in
   (b) $10,000, cash received at age 50 if living
   (c) $10,000—$9,040=$960, additional amount received, *Ans.*

3. At age 25 Mr. Lang took out a 20-year endowment policy for $3000. After having paid 7 annual premiums he died. How much more had he paid on this policy than he would have paid in the same time had he taken out a whole-life policy for the same amount when he bought the endowment policy?

   $44.79—$16.56=$28.23, difference in annual premiums per $1000
   $28.23×3×7=$592.83, additional premiums on $3000 in 7 yr.

Monthly Income Basis

Many persons prefer to buy life insurance by paying for it on a monthly basis, paying so much every month out of income. For their convenience, the monthly premium is an "even" amount, such as $5 or $10 per month, whatever their budget allows, but the face value of the policy, instead of a round sum like $2000 or $3000, is an odd amount, such as $3236, etc.

EXAMPLES

1. Using the above table, find how much 20-payment life insurance can be purchased for $10 a month at age 23? What annual rate per $1000 does this amount to?

4. Mr. Shuster is undecided as to which kind of policy he should buy. If he is 25 years old, how much more per year will a \$4000 20-year endowment policy cost him than a 20-payment life policy for the same amount? (b) How much more will he have paid at the end of twenty years if he buys the endowment policy?

   (a) \$44.79—\$26.42=\$18.37
   \$18.37×4=\$73.48 extra per year

   (b) \$73.48×20=\$1469.60, total extra cost

Policy Privileges

After premiums have been paid on life insurance policies for a certain period, generally from two to three years, the insured is entitled to certain privileges thereafter, as for example:

(1) The privilege of surrendering the policy for a stipulated cash payment, thereby terminating the insurance. This cash payment is called the *cash surrender value,* and increases the longer the policy has been "in force".

(2) The privilege of discontinuing premium payments and accepting a smaller amount of insurance (reduced benefit) for the duration of the policy. The policy is then treated as a "paid-up" policy, and the reduced benefit is called the *paid-up insurance.*

(3) The privilege of discontinuing premium payments and accepting, instead of a reduced benefit, the same face value amount of insurance for a specified, limited number of years, after which time, if the insured is still living, the policy automatically terminates. This is called *extended term insurance.*

(4) The privilege of borrowing money from the company, up to an amount equal to the cash surrender value of the policy at the time the loan is made. The company, when lending the insured money, charges interest at from 5% to 6% per year, and takes the policy as collateral for the loan. If the loan is not repaid during the life of the insured, then the beneficiary receives the face value of the policy less the amount of the loan together with unpaid interest.

$5050, *Ans.*
$10×12=$120 per year
$120÷5.05=$23.76, *Ans.*

2. How much 20-year endowment insurance can be bought on this basis at age 41 by paying $180 a year in monthly installments?

$180 per year=$15 per month at age 41,
$10 per month purchases $2538
and $ 5 " " " 1269
$3807, *Ans.*

Annuities

Purchasing an annuity is a method whereby an individual by parting with a given principal may in return receive an income guaranteed for the remainder of his life. It removes the worry of loss or depletion of principal and substitutes the satisfaction of a fixed and regular income for the remainder of the annuitant's life. The three most general forms of annuities are:

(1) *Deferred annuity,* purchased either by a series of payments, or by a lump sum, but where the income does not begin until some future date, perhaps at age 55, 60, or 65. Usually, it provides for the return of the premiums paid plus interest in the event of surrender or death before the income begins. Such a contract appeals particularly to persons who have no dependents, or who have adequate insurance protection and are now chiefly concerned with provision for their old age.

(2) *Immediate annuity,* purchased by a lump sum where the income begins immediately. Under such a contract, the income may be guaranteed for life only, ceasing with death; or, a beneficiary may be named who at the annuitant's death will receive in installments the difference, if any, between the purchase price paid and the sum of the installments received by the annuitant. In the latter case, of course, the income is somewhat less than when the income is guaranteed for life only.

AMOUNT OF INSURANCE PURCHASED BY A MONTHLY PREMIUM OF $10			
	Whole Life	*20-Payment Life*	*20-Year Endowment*
Age	*Amount of Insurance*	*Amount of Insurance*	*Amount of Insurance*
16	$9174	$5747	$2739
17	9009	5649	2732
18	8849	5555	2732
19	8620	5434	2732
20	8474	5347	2724
21	8333	5263	2724
22	8130	5154	2724
23	7874	5050	2717
24	7692	4950	2717
25	7462	4878	2717
26	7299	4761	2710
27	7092	4672	2710
28	6896	4587	2702
29	6666	4484	2702
30	6493	4385	2695
31	6289	4291	2688
32	6060	4201	2680
33	5882	4098	2666
34	5681	4000	2659
35	5494	3906	2645
36	5291	3802	2631
37	5102	3703	2617
38	4926	3610	2597
39	4739	3521	2583
40	4566	3424	2557
41	4385	3333	2538
42	4219	3236	2512
43	4048	3144	2487
44	3891	3058	2457
45	3731	2967	2427

III. *For temporary total disability:*
Two-thirds of the average weekly wage during the period of disability, but not to exceed \$5000.

IV. *For temporary partial disability:*
If earning capacity is decreased, two-thirds of the difference between his average weekly wage before the accident and his wage-earning capacity thereafter, but not to exceed \$4000.

Premium Payments

Premiums are paid annually to the company by the employer. They are based in all cases on a pay roll of \$100. The rates vary with the nature of the workman's job, ranging from 10¢ per \$100 pay roll for clerical workers to \$5.13 per \$100 pay roll for carpenters in a shop or factory.

Hospitalization Plans

At the present time there are about 50 non-profit hospital-service plans operative in the United States. They are frequently referred to as "hospitalization associations", the "three-cents-a-day plan for hospital care", and so on. Typical premiums range from \$10 to \$12 annually per adult, and from \$26 to \$30 annually per family group. If payable quarterly the rate is a little higher. An annual family group premium of \$26 would cost \$6.60 quarterly. The chief benefit received is the privilege of occupying a semi-private room, in any of the hospitals belonging to the association, for a maximum period of 21 days during any given year, at a saving to the subscriber of about \$4.50 for each day that he occupies the room. In addition, certain other minor services are also usually furnished.

SOCIAL SECURITY

Social Security Act

This law was first enacted in August, 1935, and is designed to help persons whose annual wages are under \$3000. The data given here are in accordance with an amendment signed

The premiums on this policy for men are:

Class A risks: $23.30
Class B risks: $28.75
- - - - - - - - - - - - - - -
Class D risks: $44.15

Accidents while Traveling

Many companies also issue accident insurance while traveling, or "accident ticket policies". Thus, for a premium payment of 25¢ a day while on a trip, one company will pay up to $5000 for accidents occurring on public carriers, and up to $3000 for other accidents, as well as certain weekly indemnities.

Workmen's Compensation

This is a form of protection against industrial accidents affecting workmen. In New York State, for example, any employer hiring four or more persons, whether the occupation is hazardous or not, is held liable for compensation to his workers if injured accidentally while on the job. Teachers, ministers, farm laborers, and domestic servants are exempted; all state employees are entitled to compensation, but municipal employees are entitled to compensation only if the occupation is listed as "hazardous".

Benefits Prescribed by Law

The benefits to which a workman is entitled by the Workmen's Compensation Law are as follows:

I. *For permanent total disability:*
Two-thirds of the average weekly wage during the period of disability, with a maximum of $100 per month, and all medical expenses paid.

II. *For permanent partial disability:*
Two-thirds of the average weekly wage for various periods from 15 weeks up to 312 weeks, depending upon the nature of the injury sustained.

For convenience, the following table has been prepared, showing the retirement benefits for various incomes and ages of entry. Study of the table will reveal that the lower paid and older employees are favored by the plan; for example, at age 25 of entry, a worker receiving $250 a month, or 5 times the wages of one earning $50, receives only *twice* as great a benefit ($56 instead of $28).

MONTHLY RETIREMENT BENEFITS, BEGINNING AT AGE 65

Average Monthly Earnings	*$50*	*$100*	*$150*	*$200*	*$250*
Age at entry					
25	$28.00	$35.00	$42.00	$49.00	$56.00
35	26.00	32.50	39.00	45.50	52.00
45	24.00	30.00	36.00	42.00	48.00
55	22.00	27.50	33.00	38.50	44.00
60	21.00	26.25	31.50	36.75	42.00

Benefits to Dependents and Survivors

In addition to the primary benefit payable to employees upon retirement, the Social Security Act also provides for payments to the dependents of a living pensioner, to the survivors of a deceased pensioner, or to the survivors of an insured employee. All of these benefits are based upon the amount of the primary benefit calculated as of date of death or retirement, and are subject to certain qualifying conditions which cannot be given in detail here. The table shows, in general, the amount of the dependent's benefit and when it becomes effective.

Maximum and Minimum Limits

Whenever the total monthly payments payable to an employee and his dependents, if any, exceed $20, the law provides

in August, 1939, which became effective January 1, 1940. Until 1943 both the employer and the employee each contribute 1% of the employee's wages; beginning with 1943 and until 1946, their contributions are to be 2% each, from 1946 to 1949, 2½%, and from 1949 on 3% for both employer and employee. These contributions are accumulated in a special trust fund from which all benefits are paid.

Monthly Benefits

Under this amendment the benefits payable are based upon "average taxable wages" instead of total wages. In computing "average wages", the total taxable wages earned are divided by the total number of months in which wages could have been earned whether or not the employee actually worked.

The "primary benefit" is a monthly benefit payable to an employee who has attained age 65, who is actually retired, and who otherwise qualifies through having been insured in accordance with the terms of the act. This primary benefit is the base from which benefits to wives, widows, dependent children and parents are figured.

Computing the Monthly Benefit

According to the terms of the act, the amount of the monthly benefit is computed by adding (a), (b), and (c):

(a) 40% of the first $50 of average monthly wages;
(b) 10% of the next $200 of average monthly wages;
(c) 1% of (a)+(b), multiplied by the number of *years* in which taxable earnings were $200 or more.

EXAMPLE

Mr. Ferguson has been receiving $125 a month for 15 years. What is the amount of his monthly benefit?

(a) 40% of $50	=$20.00
(b) 10% of the remaining $75=	7.50
(c) 1% of (a)+(b), times 15=	4.13
Primary benefit	=$31.63

CHAPTER XXIII

MATHEMATICS OF INSURANCE: PROPERTY PROTECTION

We have just seen how people can provide for personal security through insurance. Now we shall learn how property and other investments may be protected against damages or loss due to accidents, the elements, or other uncontrollable events—"acts of God", as the legal insurance phrase has it. Such hazards include primarily fire, burglary and theft, public liability, and collision. Most commonly insured are houses, household belongings, automobiles, business premises, and merchandise.

Protection may also be secured against earthquake, tornado, hail, windstorm, lightning and flood; against marine disaster; against boiler explosions; against breakage of plate glass and machinery; against damage due to airplane crashes; and against loss or injury of crops and livestock.

These modern developments of property insurance are outgrowths of a crude form of marine insurance in the form of "loans on bottomry" during the Middle Ages, and of early fire insurance which began in England some 250 years ago. In our own country, fire insurance was already fairly well begun even in pre-revolutionary days.

Relationship	*Amt. of Benefit*	*Becomes effective*
Wife	½ Primary Benefit	When wife is age 65 and employee is retired.
Child	½ Primary Benefit	Up to age 18 if dependent. (Applicable to child of living pensioner as well as to child of deceased employee).
Widow	¾ Primary Benefit	At age 65; also while any surviving dependent child is under 18.
Parent of Deceased Employee	½ Primary Benefit to Each	At age 65 if no widow and no unmarried child under the age of 18 and if parent is dependent.

that the total amount payable shall not be more than whichever of the following is smallest:

(a) \$85 per month
(b) 200% of the primary benefit
(c) 80% of the insured's average monthly wage

If the total monthly benefits payable are less than \$10, such benefits shall automatically be increased to \$10.

for example, a store located next to a dyeing and cleaning establishment.

(4) *PROTECTION*. The rate diminishes as the facilities for fire prevention and fire fighting increase; these include proximity to fire hydrants, adequacy of local fire department, and equipment with automatic sprinklers, fire extinguishers and other safety devices.

Classification and Rates

Cities and towns are generally classified into ten groups numbered 1, 1½, 2, 2½, 3, 3½, and so on to 6, depending upon the degree of fire protection afforded; the higher the number of the class, the higher the rate. Individual rate books are prepared for each city by a licensed inspection bureau. These show the rates on property in individual cities. The rates are the same for all companies. In determining the rate for a particular building, the inspection bureau starts with a so-called *basic rate,* to which are further additional charges to cover items which increase the hazard, and from which there are subtracted items which diminish the likelihood of fire loss. The rate for any building is fixed by an actual inspection of the property and the application of insurance rules regarding basic rates, exposure, additional charges, and credit.

Premium Rates

The amount of the premium, unless otherwise specified, covers insurance for one year. The stated premium is an annual rate. It is based on $100 worth of protection. For example, if the rate on a certain building is 42¢ per $100, and the building is insured for $2500, the annual premium to be paid equals 25×$.42, or $10.50.

EXAMPLES

1. Mrs. Halliday insured her household belongings for $1800, paying an annual premium rate of 24¢ per $100. What is the amount of the annual premium?

FIRE INSURANCE

This is undoubtedly the most widely used form of property insurance. No businessman would be without fire insurance; many business houses refuse to sell merchandise on credit to individuals whose property is not adequately protected by fire insurance. Likewise, no one should be without fire protection on his house as well as his furniture and household belongings. A mortgage cannot be secured on a house unless the building is adequately covered against fire loss.

Fire insurance policies generally cover all losses caused by fire, directly or indirectly, excluding loss by lightning unless the building catches fire. The protection extends also to physical damage caused by water or chemicals used in extinguishing a fire; spoilage due to smoke; or mechanical damage, wreckage, etc., caused by the firemen's efforts.

Fire Hazard

The premium rates charged for fire insurance are determined statistically, and depend upon what is technically known as the *fire hazard*. In other words, the rates depend upon such considerations as the size and location of the building, the nature of its construction, the purposes for which it is used, and the available protection against fire. Specifically, the chief factors entering into rate determination are:

(1) *KIND OF BUILDING.* Insurance on brick houses costs less than on wooden or frame houses; if the building is "fireproof", i.e., steel and concrete, etc., the rate is still lower. Also, the higher the building, the higher the premium rate, since it is more difficult to extinguish fires near the top.

(2) *OCCUPANCY.* The rate also depends upon what the building is used for. A factory using chemicals, paints, or other inflammable materials such as paper or wood shavings will carry a higher rate than an office building or a store.

(3) *EXPOSURE.* The rate depends also upon the proximity of the building to other buildings which might take fire readily, as,

4 years at 28¢ per $100 annually. Find (a) the total premium; (b) the total yearly cost of fire protection.

$1\frac{3}{4} \times \$.20 = \$.35$
$\$.35 \times 72 = \$25.20$

$1\frac{3}{4} \times \$.28 = \$.49$
$\$.49 \times 25 = \$12.25$

$\$25.20 + \$12.25 = \$37.45$, total premium, *Ans.* (a)
$\$37.45 \div 2 = \$18.73$, total yearly cost, *Ans.* (b)

3. A business firm insured its stock of merchandise for 80% of its value for a 5-yr. period at an annual rate of 44¢ per $100. If the inventory value of the stock is $40,000, what is the annual insurance cost that must be included in the overhead?

$\$40{,}000 \times 80\% = \$32{,}000$
$320 \times 4 \times \$.44 = \$563.20$

$\$563.20 \div 5 = \$112.64$, yearly cost of insurance, *Ans.*

Fire Losses Paid

No fire insurance company ever pays a policyholder more than the actual loss due to fire. The exact amount paid is determined by an *insurance adjuster,* who represents the company, inspects the damage, and estimates the amount of the loss or the cost of replacement. The claim is then settled to the mutual satisfaction of the company and the policyholder.

Since most cities and towns have reasonably adequate fire protection, the *total* destruction by fire of a house or a store is the exception rather than the rule. For this reason, the average owner prefers to insure his property only for *part of its value* rather than the full value. In order to prevent the owner from insuring for *too small* a part of the value, therefore, the insurance company in almost all cases includes an "80% clause"* in the policy, whereby the company will pay only a part of the loss unless the property has been insured for at

*Strictly speaking, this refers to the "New York Standard Average Clause," although it is commonly called a "coinsurance" clause, even by the courts. Other percentages than this are sometimes used, but 80% is by far the most common.

$1800÷$100=18
18×$.24=$4.32, *Ans.*

2. A house is insured for 80% of its value to protect a mortgage-holder. If the house is valued at $9,600 and the annual rate is $.40 per $100, what is the annual premium?

$9600×80%=$7680
$7680÷$100=76.8
76.8×$.40=$30.72, *Ans.*

Term Rates

Fire insurance policies are usually issued for a period of two, three, four, or five years. Fire insurance written for a period greater or less than a year is called *term* insurance. If the period is greater than a year, the rate charged is computed by adding 75% of the annual rate for each additional year. For example:

RATES—TERM INSURANCE

Period	*Rate*
2 years	1¾ times the annual rate
3 "	2½ " " " "
4 "	3¼ " " " "
5 "	4 " " " "

EXAMPLES

1. A home owner insures his house for $6500 for 3 years at an annual rate of 24¢ per $100. Find (a) the premium; (b) the yearly cost of his insurance.

Annual rate=$.24 per $100
Rate for 3 yr.=2½×$.24=$.60

(a) Premium=$.60×65=$39, *Ans.*
(b) Yearly cost=$39÷3=$13, *Ans.*

2. Mr. Fairchild insured his house for $7200 for 2 years at an annual rate of 20¢ per $100 and his household belongings for $2500 for

2. A house worth \$9000 is insured for \$6000 under an 80% clause. If the fire loss amounts to \$1800, how much will the company pay?

$$\frac{\text{Face of policy}}{80\% \text{ of value}} \times (\text{loss}) = \frac{\$6000}{\$7200} \times \$1800 =$$
$$\tfrac{5}{6} \times \$1800 = \$1500, \textit{ Ans.}$$

3. A store and its contents worth \$25,000 are insured for \$12,000. Loss by fire amounts to \$12,000. How much will the company pay?

$$\frac{\text{Face of policy}}{80\% \text{ of value}} \times (\text{loss}) = \frac{\$12{,}000}{\tfrac{4}{5} \times 25{,}000} \times \$12{,}000 =$$
$$\tfrac{12}{20} \times \$12{,}000 = \$7{,}200, \textit{ Ans.}$$

Distribution of Risk

Insurance companies do not usually risk large amounts on any one location. If a building, for example, is to be insured for \$150,000, this insurance would be split into several smaller policies placed with different companies; thus Company A might take \$40,000; Company B, \$50,000; and Company C, pay $\frac{40{,}000}{150{,}000}$, or $\frac{4}{15}$ of the \$30,000 loss; Company B would pay $\frac{50{,}000}{150{,}000}$, or $\frac{1}{3}$ of \$30,000; and Company C would pay $\frac{60{,}000}{150{,}000}$, or $\frac{2}{5}$ of \$30,000. In other words, each company pays its *pro rata* share of the loss, i.e., a share proportionate to the amount of risk assumed.

Another way of distributing the risk is for a company to issue a policy for \$60,000, and then to "reinsure" in its own favor for \$35,000 with some other company. By this procedure of reinsurance, the company issuing the original policy for \$60,000 actually assumes a risk of only \$25,000.

least 80% of its value. In other words, if the insured (under an 80%-"coinsurance clause") wishes to obtain the full amount of the loss, he must carry an amount of insurance equal to 80% of the value of the property. This principle may be better understood by studying the following rule:

$$\left\{\begin{matrix}\text{Claim paid by}\\ \text{the company}\end{matrix}\right\}=\left(\frac{\text{Insurance carried}}{80\%\text{ of value of property}}\right)\times(\text{fire loss}).$$

Some of the various situations that might arise and how they would be met under an 80%-coinsurance clause are given in the following table; remember that the company never pays more than the face of the policy, nor more than the amount of the loss. Whenever the loss is equal to or greater than 80% of the value, the clause is without effect, and the company pays up to its policy limit.

HOW THE 80%-COINSURANCE CLAUSE WORKS

Case	*Condition*	*Value of Property*	*Face of Policy*	*Actual Loss*	*Amount Paid*
(1)	Face of policy=80% of value or more	$5000	$4000	$1500	$1500
(2)		5000	4000	4000	4000
(3)		5000	4000	4800	4000
(4)		5000	4500	4200	4200
(5)		5000	4500	4800	4500
(6)		5000	5000	5000	5000
(7)	Face of policy is less than 80% of value	5000	3000	2400	1800
(8)		5000	2400	2400	1440
(9)		5000	2000	2400	1200

EXAMPLES

1. Property worth $10,000 is insured for $8000 under an 80% clause. How much will be paid if the loss amounts to $750?

$$\frac{\text{Face of policy}}{80\%\text{ of value}}=\frac{\$8000}{\$8000};\ 1\times\$750=\$750,\ \textit{Ans.}$$

Maximum Limits	*Per Cent of Minimum Premium*	*Illustrations*
5/10,000	100%	$32.00
10/20,000	115%	$36.80
25/50,000	127%	$40.64
50/100,000	132%	$42.24

Property Damage

This is usually written up to $5000 maximum; typical rates for such a policy would be:

in New York City $12.70
in Kingston, N. Y. ....$ 7.00

EXAMPLE

Mr. Flynn wishes to insure himself against public liability for $25,000/$50,000 and against property damage for $5000 in a community where the minimum P. L. rate is $34.50 and the P. D. rate is $9.75. What is the combined annual premium for this protection?

$34.50×127%=$43.82, public liability premium
$43.82+$9.75=$53.57, total premium, *Ans.*

Comprehensive Damage and Collision

The cost of insurance against damage done to one's own car, whether through collision or not, depends, in addition to the type of community in which the owner lives, upon the *original price* of the car, as well as upon its *age*. In other words, the value of the car, including its initial cost and the amount of depreciation, plays an important role in determining the premium rates. Typical rates for a moderately priced car are given herewith.

EXAMPLE

Mr. Adams, a resident of Kingston, N. Y., owns a car which he bought new 15 months ago for $960. He insures it against public

AUTOMOBILE INSURANCE

Extent of Coverage

Automobile insurance may be obtained to protect the owner in four ways: (1) *Public Liability,* which reimburses the owner for costs or damage suits arising from injuries to the public; (2) *Property Damage,* which covers the cost of damage done to the property of others, including another's car, as well as any other property damage; (3) *Comprehensive Material Damage,* which refers to any loss of, or any damage to one's own car, except by collision, and includes fire, theft, windstorm, hail, lightning, and plate glass insurance; and (4) *Collision Insurance,* which covers damage sustained by one's own car as a result of a collision.

Public Liability

This coverage is expressed as \$5000/\$10,000, which means that a maximum limit of \$5000 will be paid for injuries to any one person, or \$10,000 total will be paid for any one accident; similarly, a policy may guarantee \$10,000/\$20,000, or 25/50,000, or 50/100,000.

Naturally, the premium rates will depend not only upon the amount of liability covered, but also upon the relative likelihood of accidents occurring, which in turn depends upon the neighborhood in which the car is chiefly operated. Rates in large traffic-laden cities, as might be expected, are considerably higher than in small cities and towns that have a much smaller density of traffic. For example, typical rates for 5/10,000 would be:

in New York City.............\$53
in Kingston, N. Y.............\$24

Furthermore, as the amount of such coverage increases, the rates do not increase proportionately, but rise instead according to the following scheme:

CHAPTER XXIV

MATHEMATICS OF TAXES

THE BASIC PURPOSE of all taxation, at least in a democracy, is to secure the necessary money to pay for the cost of public service and social benefits provided for the people by governmental agencies. Such public service includes police and fire protection, playgrounds and parks, water supply, sanitation and public health, hospitals, prisons, public schools, libraries and museums, bridges and highways—to mention but a few which are primarily the concern of local communities, such as cities, towns and counties.

In recent years, state governments and particularly the Federal government, have extended and provided many new benefits of service to the public, including, for example, farm loans, home owner's loans, Federal housing, social security, unemployment insurance and a number of others, to say nothing of the traditional services such as providing military protection, navigational service, weather bureau, agricultural experimentation, forestry patrol, crop preservation against pests, conservation of natural resources, etc.

All of these services cost money, and in this final chapter we shall note briefly a few of the sources of government income. All taxes fall roughly into five categories, viz., (1) prop-

	Comprehensive Material Damage	*Collision ($50—deductible)*
New York City	$1.60 per $100 new car value $1.75 after 18 mo. car age	*Class of Car:* A .... $32.00 B .... 36.00 C .... 41.00 D 46.00
Kingston, N. Y.	$.70 per $100 new car value $.85 after 18 mo. car age	*Class of Car:* A .... $19.00 B .... 21.00 C .... 24.00 D 27.00

liability for 10/20,000; for $5000 property damage; also for comprehensive material damage and for $50-deductible collision damages. What is the combined premium for one year (Class B car)?

$24×115%=$27.60, Public Liability
7.00, Property Damage
9.6×$.70 = 6.72, Comprehensive Material Damage
21.00, Collision ($50 deductible)

$62.32, Total Premium, *Ans.*

Combined Duties

Certain commodities carry a combined ad valorem and a specific duty. In this case the total duty is computed as shown below.

EXAMPLE

What is the duty on a shipment of 1½ tons of aluminum ware worth \$4200, if the duty is 8½¢ per lb. and 40% ad valorem?

1½ tons=3000 lb.

\$.085×3000=\$ 255
\$4200×.40=\$1680
\$255+\$1680=\$1935, *Ans.*

INCOME TAXES

Taxes based upon income rather than tangible property or commodities make use of the principle of the ability to pay, i.e., individuals or business firms having higher incomes pay a larger tax than those with lower incomes. Both the Federal government as well as most of the state governments impose an income tax.

Net Income and Exemptions

The rates and exemptions on income tax are subject to change from time to time, as prescribed by law. We shall merely give typical instances, reminding the reader that they do not necessarily agree with current regulations.

A person is rarely required to pay a tax based upon his entire income; certain exemptions and deductions are permissible. "Earned income" refers to wages, salary, professional fees and any money received for personal services actually rendered. "Unearned income" refers to interest received from loans or bank accounts; interest and profits from stocks and bonds; rents; and royalties from patents, publications, etc.

erty taxes; (2) customs; (3) income taxes; (4) excise taxes, including sales taxes; and (5) miscellaneous and special taxes. Property taxes have already been discussed in Chapter XI.

CUSTOMS TAXES

Import Duties

The Federal government collects a tax on many articles and commodities that are imported from foreign countries. These taxes are called *customs taxes,* or *duties.* Articles on which no such tariff is imposed are called duty-free. Two kinds of duties are charged: (1) *ad valorem* duty, and (2) *specific* duty.

Ad Valorem Duty

This means that the merchandise is taxed at a certain per cent of its net value. An ad valorem duty is not charged on fractions of a dollar; 50¢ or more is considered an additional dollar, and less than 50¢ is disregarded.

EXAMPLE

Find the ad valorem duty on 24 boxes of toilet soap valued at \$3.95 each, if the duty is 30%.

$\$3.95 \times 24 = \$94.80$
$\$95 \times .30 = \$28.50$, *Ans.*

Specific Duty

This is a tax calculated on the basis of the quantity of goods imported, regardless of their cost or value. The duty is expressed as so much per pound, per yard, per ton, etc. Fractions of one half a unit or more are considered as an additional whole unit; less than half a unit is disregarded.

EXAMPLE

What is the duty on a shipment of 2 tons of Brazil nuts at 1½¢ per lb.?

$2000 \times 2 = 4000$ lb.
$\$.015 \times 4000 = \$60$, *Ans.*

Salary		$4400
Less personal exemption	$1500	
Less exemption for dependents	800	2300
		$2100
Less earned income credit (1/10 of $2100)		210
		$1890
Income from investments		$ 450
Income from rents		125
Net taxable income		$2465
Less deductions:		
Charities	$ 25	
Taxes	312	
Professional expenses	75	
Net income subject to tax		$2053

$2053 \times .06 = \$123.18$, *Ans.*

SPECIAL TAXES

Occupancy Tax

In some communities, as in New York City for example, the municipal government levies an *occupancy tax,* which is payable by persons occupying rented premises for a gainful purpose. Some of these classes of occupations are:

Accountants
Advertising Signs
Architects
Beauty Parlors
Brokers
Carpenters
Chiropodists
Chiropractors
Coal Dealers
Doctors and Dentists
Dressmakers
Electricians
Furnished Rooming Houses
Lawyers
Locksmiths
Milliners
Musical Instructors
Notaries
Painters
Plumbers
Public Stenographers
Registered Nurses
Service Repairmen
Veterinarians

Basic exemptions in recent years have run as follows:

(1) Unmarried persons, not heads of households ....$ 750
(2) Head of a family $1500

Allowances might run:

(1) A personal credit for each dependent, other than husband or wife ....$ 400
(2) An "earned-income credit" of $\frac{1}{10}$ of the earned income

Deductions permitted often include:

(1) Interest on borrowed money.
(2) Contributions to churches and organized charities.
(3) Losses from bad debts.
(4) Certain taxes that have been directly levied against the individual; e.g., sales taxes, property taxes, gasoline taxes, and sometimes income taxes from other government agencies.
(5) Certain losses, such as: fire losses; automobile damages; theft; worthless securities.

Furthermore, businessmen and professional people are permitted certain deductions:

(1) Rent for premises.
(2) Depreciation of business property or professional equipment.
(3) Certain traveling and operating expenses.
(4) Fees and premiums paid in connection with workmen's compensation, employee insurance, pension funds and the like.

EXAMPLE

Mr. Ingersoll's income during a certain year was as follows: salary, \$4400; interest on investments, \$450; rental of garage, \$125. His property and other deductible taxes amounted to \$312; gifts to charity and church, \$25; professional expenses, \$75. At the rate of 6% of "net income subject to tax," what was the amount of his income tax that year, if he was the head of a family, with a wife and two young children?

per gal.? (b) what per cent of the total price is the tax? (c) if the motorist's average yearly consumption of gasoline is 600 gal., how much does he pay annually in gasoline taxes?

(a) $\$.15+\$.0375=\$.18\frac{3}{4}$, cost per gallon, *Ans.*
(b) $3\frac{3}{4}¢\div18\frac{3}{4}¢=\frac{1}{5}=20\%$, *Ans.*
(c) $600\times\$.0375=\$22.50$, *Ans.*

Toll Charges

Another form of taxation is used to raise funds to pay the cost of bridges and highway construction. These taxes are known as *toll charges,* and are payable only by motorists who use the roads and bridges. The table below is typical.

TOLL SCHEDULE

Rip Van Winkle and
Mid-Hudson Bridges

	Axles	
Passenger Auto		\$ .50
Passenger Auto with Trailer		.75
Trucks — 2 tons or under	2	.50
" — over 2 tons, inc. 5 tons	2	.75
" — 2 tons or under	3	.75
" — over 2 tons, inc. 5 tons	3	1.00
" — over 5 tons	2	1.00
Bus	2	.60
Extra axles on all vehicles		.25

No Extra Charge for Passengers

Commutation Books

Passenger Autos			
10 trips	6 months		\$ 3.50
60 trips	50 days		12.00
50 trips	1 year		15.00
Trucks	Rated Capacity		
25 trips	2 tons or under	6 months	10.00
50 trips	over 2 tons, inc. 5 tons		30.00

Rates subject to revision at discretion of N. Y. S. Bridge Authority.

In these instances the *owner* of the premises is not taxable, but only the tenant who uses the rented premises for conducting a business or making a living.

Licenses and Fees

Federal, state and local governments also levy imposts on a miscellany of items, such as marriage licenses, hunting and fishing licenses, peddlers' licenses, automobile licenses; also taxes on legal documents, deeds, mortgages, notes; transfer taxes on stocks and bonds; liquor licenses; excise taxes on liquor, cigarettes, playing cards; sales taxes; etc. Many of these taxes are collected by means of stamps pasted on the documents or commodity packages.

Inheritance and Estate Taxes

A tax on property, real and personal, left as a legacy upon an individual's decease, is subject both to a Federal *estate tax* and a state *inheritance tax*. The rate varies from time to time, from state to state, and with the nearness of the relatives who share in the inheritance.

EXAMPLES

1. The motor vehicle bureau of a certain state charges a fee when issuing license plates. If the tax rate is 75¢ per 100 lb. or fraction thereof, what is the fee for an automobile weighing 2450 lb.?

   25×\$.75=\$18.75, *Ans.*

2. If the Federal tax on theater tickets is 10%, what is the total cost of a movie theater ticket if the price of admission is 35¢? If a couple attend the movies on the average 5 times a month, how much do they pay a year in taxes on movie tickets?

   35¢×10%=\$.035=\$.04
   \$.35+.\$04=\$.39, price per ticket.
   2×5×12×\$.04=\$4.80, yearly tax, *Ans.*

3. In a certain state the tax on gasoline is 3¾¢ per gallon. If the cost of the gasoline is 15¢ per gal., (a) what must the motorist pay

No. 8

1. 124,969 *2.* 151,843 *3.* 164,296
4. 72,574 *5.* $2160.28 *6.* $7753.84
7. $379,536.55 *8.* $606,814.81
9. 151,478 *10.* $437,580,078

No. 9

1.	2.	3.
356	508	537
114	239	262
287	263	238

4.	5.
$287.14	$ 80.66
462.42	76.53
530.63	85.51
328.12	159.75
395.26	47.15
	104.49

No. 10

1. 23,718 *2.* 129,485 *3.* 148,208
4. 387,296 *5.* 563,640 *6.* 1,614,732
7. 8,120,736 *8.* $11,498.50
9. $20,691 *10.* $9906.10

No. 11

1. $24 *2.* $30 *3.* $86
4. $90 *5.* $200 *6.* $3600
7. $2700 *8.* $12,000 *9.* $1800
10. $149.80 *11.* $1.08 *12.* $864.50

No. 12

1. $1330\frac{4}{7}$ *2.* $959\frac{1}{9}$ *3.* $5741\frac{1}{2}$
4. $6027\frac{7}{11}$ *5.* $62,334\frac{1}{2}$ *6.* $71,107\frac{7}{12}$
7. $108\frac{41}{43}$ *8.* $1667\frac{9}{19}$ *9.* $233\frac{59}{328}$
10. $3462\frac{19}{24}$

ANSWERS

No. 1

1. 59	*2.* 61	*3.* 59	*4.* 68	*5.* 57
6. 50	*7.* 67	*8.* 51	*9.* 57	*10.* 59

No. 2

1. 413	*2.* 231	*3.* 433	*4.* 278	*5.* 432
6. 493	*7.* 570	*8.* 570	*9.* 532	*10.* 516

No. 3

1. 4254	*2.* 4949	*3.* 43,240	*4.* 38,275
5. $249.22	*6.* $2664.05	*7.* 43,942	
8. $4808.40	*9.* $1608.17	*10.* $119,687.74	

No. 4

1. $730.74	*2.* 660,025	*3.* $31,862.50	*4.* $591,944.68

No. 5

1.	*2.*	*3.*	*4.*	*5.*
22	22	27	142	$112.52
18	17	27	170	179.99
18	26	23	114	125.22
18	22	28	142	152.17
76	87	105	214	$569.90
			782	

No. 6

1. 22, 42, 10, 22, 25, 53, 24, 25
2. 345, 321, 171, 22, 311, 561
3. 611, 342, 402, 134, 220, 282

No. 7

1. 72, 62, 84, 97, 84, 82
2. 522, 932, 305, 891, 830
3. 642, 925, 923, 403, 623

No. 20

1. \$5000
2. \$900
3. \$4200
4. \$800
5. \$28,536
6. .027 oz.
7. 13.8%
8. \$520
9. \$45,000
10. \$35,000

No. 21

1. 18″
2. $3\frac{1}{8}'' \times 7''$
3. 12%; 89.3%
4. \$201.60
5. \$8000
6. $16\frac{2}{3}\%$; 7:3
7. $2\frac{1}{4}''$
8. $312\frac{1}{2}$ ft.

No. 22

1. 7.48 gal.
2. $8\frac{1}{3}$ lb.
3. \$7.70
4. 13.5 kg.
5. \$17.05; \$1.07
6. 418 gm.

No. 23

1. (a) 38; (b) 13; (c) 110
2. (a) 7m; (b) M+10; (c) H—4; (d) 3k cents; (e) 24p dollars
3. (a) P/3=20; (b) $\frac{S}{5}=30$; (c) $\frac{5w}{2}=40$; (d) $x-4=12$; (e) 2d+8=100
4. (a) 217,000,000; (b) .0000018; (c) 459,000,000
5. (a) 84.3; (b) 13.6; (c) 24.3; (d) 56.2

No. 24

1. 15
2. \$15
3. $P=\frac{I}{RT}$; $R=\frac{I}{PT}$
4. $R=\frac{L}{2\pi h}$
5. 30,000 ft. lb.
6. 6
7. 5
8. $\frac{E-IR}{I}$
9. $\frac{2A}{h}-b$
10. (a) tripled; (b) one-fourth as large; (c) six times as large

No. 25

1. 1066.5 sq. in.
2. 68.4 cu. in.
3. $27\frac{1}{2}$ sq. in.
4. 27 in.
5. 126.9 ft.
6. 56 ft.
7. 16
8. 21.4%

No. 26

1. 63.0 ft.
2. 666 yd.
3. 10.6 ft.
4. 8914 ft.
5. 42.9 ft.; 69.8 ft.

No. 13

1. 53	2. 19.4	3. 45.4
4. 61.17	5. 73.44	6. 483.93
7. 35.9 mi./hr.	8. 19.4 sq. ft.	9. 2.42 in.
10. $137.07		

No. 14

1. $3\frac{2}{3}$	2. $1\frac{13}{14}$	3. $1\frac{1}{5}$	4. $2\frac{1}{3}$
5. $3\frac{1}{7}$	6. $2\frac{8}{15}$	7. $3\frac{1}{6}$	8. $1\frac{3}{4}$
9. $\frac{43}{4}$	10. $\frac{65}{4}$	11. $\frac{45}{2}$	12. $\frac{58}{5}$
13. $\frac{62}{5}$	14. $\frac{47}{8}$	15. $\frac{51}{16}$	16. $\frac{155}{6}$

No. 15

1. $1\frac{1}{24}$	2. $1\frac{3}{20}$	3. $1\frac{1}{24}$	4. 2
5. $1\frac{1}{4}$	6. $1\frac{5}{8}$	7. $3\frac{1}{24}$	8. $2\frac{4}{5}$
9. $2\frac{11}{16}$	10. $28\frac{5}{8}$	11. $11\frac{1}{6}$	12. $\frac{1}{8}$
13. $\frac{3}{8}$	14. $\frac{5}{24}$	15. $\frac{6}{25}$	16. $\frac{3}{40}$
17. $5\frac{5}{6}$	18. $7\frac{1}{8}$	19. $16\frac{7}{8}$	20. $7\frac{5}{6}$

No. 16

1. $\frac{1}{8}$	2. $\frac{1}{6}$	3. $18\frac{1}{3}$	4. 129
5. $\frac{1}{9}$	6. 12	7. $\frac{1}{4}$	8. $\frac{6}{7}$
9. 14	10. 9	11. 128	12. $\frac{26}{31}$

No. 17

1. $16\frac{1}{4}$ yd.	2. 20; $3.70	3. $2\frac{1}{3}$, sugar; $1\frac{1}{2}$ milk
4. 225 lb.	5. $6	

No. 18

1. $75.84	2. 21.43%	3. 28.1%
4. 34%	5. $103.13	6. 20%
7. $33\frac{1}{3}$%	8. $328.44	9. 250
10. $3750		

No. 19

1. 1250	2. $30.63	3. $103.40
4. $2.13	5. $37.50	